

Marius Coman

**FORMULAS AND POLYNOMIALS  
WHICH GENERATE  
PRIMES AND FERMAT PSEUDOPRIMES**

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**(COLLECTED PAPERS)**

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## INTRODUCTION

To make an introduction to a book about arithmetic it is always difficult, because even most apparently simple assertions in this area of study may hide unsuspected inaccuracies, so one must always approach arithmetic with attention and care; and seriousness, because, in spite of the many games based on numbers, arithmetic is not a game. For this reason, I will avoid to do a naive and enthusiastic apology of arithmetic and also to get into a scholarly dissertation on the nature or the purpose of arithmetic. Instead of this, I will summarize this book, which brings together several articles regarding primes and Fermat pseudoprimes, submitted by the author to the preprint scientific database Research Gate.

Part One of this book, “Sequences of primes and conjectures on them”, brings together thirty-two papers regarding sequences of primes, sequences of squares of primes, sequences of certain types of semiprimes, also few types of pairs, triplets and quadruplets of primes and conjectures on all of these sequences. There are also few papers regarding possible methods to obtain large primes or very large numbers with very few prime factors, some of them based on concatenation, some of them on other arithmetic operations. It is also introduced a new notion: “Smarandache-Coman sequences of primes”, defined as “all sequences of primes obtained from the terms of Smarandache sequences using any arithmetical operation” (for instance, the sequence of primes obtained concatenating to the right with the digit one the terms of Smarandache consecutive numbers sequence).

Part Two of this book, “Sequences of Fermat pseudoprimes and conjectures on them”, brings together seventeen papers on sequences of Poulet numbers and Carmichael numbers, i.e. the Fermat pseudoprimes to base 2 and the absolute Fermat pseudoprimes, two classes of numbers that fascinated the author for long time. Among these papers there is a list of thirty-six polynomials and formulas that generate sequences of Fermat pseudoprimes.

Part Three of this book, “Prime producing quadratic polynomials”, contains three papers which list some already known such polynomials, that generate more than 20, 30 or even 40 primes in a row, and few such polynomials discovered by the author himself (in a review of records in the field of prime generating polynomials, written by Dress and Landreau, two French mathematicians well known for records in this field, review that can be found on the web adress <<http://villemin.gerard.free.fr/Wwwgvm/Premier/formule.htm>>, the author – he says this proudly, of course – is mentioned with 18 prime producing quadratic polynomials). One of the papers proposes seventeen generic formulas that may generate prime-producing quadratic polynomials.

## SUMMARY

### Part One. Sequences of primes and conjectures on them

1. Conjecture on the numbers of the form  $np^2 - np + p - 2$  where  $p$  prime
2. Conjecture on the quadruplets of primes of the form  $(p, p + 4k^2, p + 6k^2, p + 8k^2)$
3. Conjecture on the primes of the form  $(q + n)2^n + 1$  where  $q$  odd prime
4. Two conjectures on the numbers of the form  $4p^4 - 800p^2 + 5$  where  $p$  is prime
5. Three conjectures on the numbers of the form  $p(p + 4n) - 60n$  where  $p$  and  $p + 4n$  primes
6. Conjecture on an infinity of triplets of primes generated by each 3-Poulet number
7. Observation on the numbers  $4p^2 - 2p - 1$  where  $p$  and  $2p - 1$  are primes
8. Observation on the numbers  $4p^2 + 2p + 1$  where  $p$  and  $2p - 1$  are primes
9. Conjecture on the numbers  $6pq + 1$  where  $p$  and  $q$  primes and  $q = kp - k + 1$
10. Three conjectures on the numbers  $6pq + 1$  where  $p$  and  $q$  primes and  $q = 2p - 1$
11. Any square of a prime larger than 7 can be written as  $30n^2 + 60n + p$  where  $p$  prime or power of prime
12. Any square of a prime larger than 11 can be written as  $60n^2 + 90n + p$  where  $p$  prime or power of prime
13. On the numbers of the form  $pq + 10^k$  where  $p$  and  $q$  are emirps
14. Formula that generates a large amount of big primes and semiprimes *i.e.*  $529 + 60 \cdot 10^k$
15. A sequence of numbers created concatenating the digit 1 twice with a prime of the form  $6k - 1$
16. A method based on concatenation to create very large numbers with very few prime factors
17. Notable observation on the squares of primes of the form  $10k + 9$
18. Notable observation on the squares of primes of the form  $10k + 1$
19. Conjecture that states that the square of any prime can be written in a certain way
20. Conjecture on the numbers  $(p^2 - n)/(n - 1)$  where  $p$  prime
21. Conjecture on the numbers  $3p(q - 1) - 1$  where  $p$  and  $q$  are primes and  $p = q + 6$
22. Four conjectures on the numbers obtained concatenating to the right a prime with the digit 9
23. Three conjectures on the numbers obtained concatenating to the left the odd numbers with 1234
24. Conjecture on the primes obtained deconcatenating to the right the numbers  $(30k - 1)(30k + 1)$  with digit 9
25. Two formulae for obtaining primes based on the prime decomposition of the number 561
26. Four conjectures on the numbers created concatenating the product of twin primes with 11
27. Two conjectures on the numbers created concatenating an odd  $n$  with  $3n - 4$  and then with 1 or 11
28. Two unusual defined and possible infinite sequences of primes and two conjectures
29. Two conjectures on the numbers obtained concatenating the squares of two primes  $q$  and  $r$  where  $r = q + 18n$

30. Eight formulas that generate semiprimes  $pq$  such that  $n = q - p + 1$  is prime respectively  $n = p + q - 1$  is prime
31. Seven Smarandache-Coman sequences of primes
32. Two conjectures on Smarandache's divisor products sequence

## **Part Two. Sequences of Fermat pseudoprimes and conjectures on them**

1. Generic form for a probably infinite sequence of Poulet numbers *i.e.*  $2n^2 + 147n + 2701$
2. Generic form for a probably infinite sequence of Poulet numbers *i.e.*  $4n^2 + 37n + 85$
3. Two conjectures on Poulet numbers of the form  $mn^2 + 11mn - 23n + 19m - 49$
4. Three cubic polynomials that generate sequences of Poulet numbers
5. Conjecture on Poulet numbers of the form  $8mn^3 + 40n^3 + 38n^2 + 6mn^2 + mn + 11n + 1$
6. Conjecture on Poulet numbers of the form  $9mn^3 + 3n^3 - 15mn^2 + 6mn - 2n^2$
7. A list of thirty-six polynomials and formulas that generate Fermat pseudoprimes
8. A list of 15 sequences of Poulet numbers based on the multiples of the number 6
9. Bold conjecture on Fermat pseudoprimes
10. Another bold conjecture on Fermat pseudoprimes
11. Generic form of the Poulet numbers having a prime factor of the form  $30n + 23$
12. Notable observation on a property of Carmichael numbers
13. Conjecture which states that any Carmichael number can be written in a certain way
14. Sequence of Poulet numbers obtained by formula  $mn - n + 1$  where  $m$  of the form  $270k + 13$
15. Two conjectures on Super-Poulet numbers with two respectively three prime factors
16. Observation on the period of the rational number  $P/d + d/P$  where  $P$  is a 3-Poulet number and  $d$  its least prime factor
17. Conjecture on the numbers  $6(m + n) + 1$  where  $m$  and  $n$  are the two prime factors of a 2-Poulet number

## **Part Three. Prime producing quadratic polynomials**

1. A list of known root prime-generating quadratic polynomials producing more than 23 distinct primes in a row
2. Ten prime-generating quadratic polynomials
3. Seventeen generic formulas that may generate prime-producing quadratic polynomials

## Part One.

### Sequences of primes and conjectures on them

#### 1. Conjecture on the numbers of the form $np^2 - np + p - 2$ where $p$ prime

**Abstract.** In this paper I conjecture that there exist, for any  $p$  prime,  $p$  greater than or equal to 7, an infinity of positive integers  $n$  such that the number  $n^*p^2 - n^*p + p - 2$  is prime.

##### Conjecture:

There exist, for any  $p$  prime,  $p$  greater than or equal to 7, an infinity of positive integers  $n$  such that the number  $n^*p^2 - n^*p + p - 2$  is prime.

The sequence of the numbers  $n^*p^2 - n^*p + p - 2$  for  $p = 7$ :

(in other words the numbers of the form  $42^*n + 5$ )

: 47, 89, 131, 173, 215, 257, 299, 341, 383, 425 (...)

The sequence of the primes of the form  $42^*n + 5$ :

: 47, 89, 131, 173, 257, 383 (...)

Note that there are also Poulet numbers that can be written as  $42^*n + 5$ ; two of such numbers are  $341 = 11^*31$  ( $n = 8$ ) and  $8321 = 53^*157$  ( $n = 198$ ); these 2-Poulet numbers have also in common the fact that  $11^*3 - 2 = 31$  and  $53^*3 - 2 = 157$ .

The sequence of the numbers  $n^*p^2 - n^*p + p - 2$  for  $p = 11$ :

(in other words the numbers of the form  $110^*n + 9$ )

: 119, 229, 339, 449, 559, 669, 779, 889, 999, 1109 (...)

The sequence of the primes of the form  $110^*n + 9$ :

: 229, 449, 1109 (...)

The sequence of the numbers  $n^*p^2 - n^*p + p - 2$  for  $p = 13$ :

(in other words the numbers of the form  $156^*n + 11$ )

: 167, 323, 479, 635, 791, 947, 1103, 1259, 1415 (...)

The sequence of the primes of the form  $156^*n + 11$ :

: 47, 89, 131, 173, 257, 383 (...)

The sequence of the numbers  $n^*p^2 - n^*p + p - 2$  for  $p = 17$ :

(in other words the numbers of the form  $272^*n + 15$ )

: 287, 559, 831, 1103, 1375, 1647, 1919, 2191, 2463 (...)

The sequence of the primes of the form  $272^*n + 15$ :

: 1103, 3823, 4639 (...)

Note that there exist numbers that can be written in more than one way as  $n^*p^2 - n^*p + p - 2$ ; such a number is  $1103 = 7^*13^2 - 7^*13 + 13 - 2 = 4^*17^2 - 4^*17 + 17 - 2$ .

The sequence of the numbers  $n^2p - n^2 + p - 2$  for  $p = 19$ :

(in other words the numbers of the form  $342 \cdot n + 17$ )

: 359, 701, 1043, 1385, 1727, 2069, 2411, 2753, 3095 (...)

The sequence of the primes of the form  $272*n + 15$ :

: 359, 701, 2069, 2411, 2753 (...)

The sequence of the primes of the form  $110 \cdot n + 9$ , where  $n$  is of the form  $10^k$ ,  $k$  greater than or equal to 0:

: 1109, 1100009, 1100000009 (...)

The sequence of the primes of the form  $156*n + 11$ , where  $n$  is of the form  $10^k$ ,  $k$  greater than or equal to 0:

[illegible]

The sequence of the primes of the form  $342n + 17$ , where  $n$  is of the form  $10^k$ ,  $k$  greater than or equal to 0:

[illegible]

## 2. Conjecture on the quadruplets of primes of the form $(p, p+4k^2, p+6k^2, p+8k^2)$

**Abstract.** In a strict sence, the term “prime quadruplet” refers strictly to the primes  $(p, p + 2, p + 6, p + 8)$  - see Wolfram MathWorld; it is not known if there are infinitely many such prime quadruplets. In this paper I conjecture that for any  $k$  non-null positive integer there exist an infinity of quadruplets of primes of the form  $(p, p+2k^2, p+6k^2, p+8k^2)$ . Finally, I define the generalized Brun’s constant for prime quadruplets of the type showed and I estimate its value for the particular case  $k = 2$  (for  $k = 1$  the value it is known being approximately equal to 0.87).

### Conjecture:

For any  $k$  non-null positive integer there exist an infinity of quadruplets of primes of the form  $(p, p + 2k^2, p + 6k^2, p + 8k^2)$ .

### The first two quadruplets of this form for few values of $k$ :

: **for  $k = 1$**  we have  $(p, p + 2, p + 6, p + 8)$ :  
:  $(5, 7, 11, 13)$  and  $(11, 13, 17, 19)$ .

Note that, beside the first quadruplet, the rest of them must have the form  $(30n+11, 30n+13, 30n+17, 30n+19)$ .

: **for  $k = 2$**  we have  $(p, p + 8, p + 24, p + 32)$ :  
:  $(5, 13, 29, 37)$  and  $(29, 37, 53, 61)$ .

Note that, beside the first quadruplet, the rest of them must have the form  $(30n+29, 30n+37, 30n+53, 30n+61)$ .

: **for  $k = 3$**  we have  $(p, p + 18, p + 54, p + 72)$ :  
:  $(109, 127, 163, 181)$  and  $(139, 157, 193, 211)$ .

Note that all of these quadruplets must have the form  $(30n+19, 30n+37, 30n+73, 30n+91)$ .

: **for  $k = 4$**  we have  $(p, p + 32, p + 96, p + 128)$ :  
:  $(11, 43, 107, 139)$  and  $(71, 103, 167, 199)$ .

Note that all of these quadruplets must have the form  $(30n+11, 30n+43, 30n+107, 30n+139)$ .

### The first quadruplet of this form for few other values of $k$ :

: **for  $k = 5$**  we have  $(p, p + 50, p + 150, p + 200)$ :  $(131, 181, 281, 331)$ .

Note that all of these quadruplets must have the form  $(30n+11, 30n+61, 30n+161, 30n+211)$ .

: **for  $k = 6$**  we have  $(p, p + 72, p + 216, p + 288)$ :  $(101, 173, 317, 389)$ .

Note that all of these quadruplets must have the form  $(30n+11, 30n+83, 30n+227, 30n+299)$ .

: **for  $k = 7$**  we have  $(p, p + 98, p + 294, p + 392)$ :  $(269, 367, 563, 661)$ .

Note that all of these quadruplets must have the form  $(30n+29, 30n+127, 30n+323, 30n+421)$ .

: **for  $k = 8$**  we have  $(p, p + 128, p + 384, p + 512)$ :  $(179, 307, 563, 691)$ .

Note that all of these quadruplets must have the form  $(30n+29, 30n+157, 30n+413, 30n+541)$ .

: **for  $k = 9$**  we have  $(p, p + 162, p + 486, p + 648)$ :  $(71, 233, 557, 719)$ .

Note that all of these quadruplets must have the form  $(30n+11, 30n+173, 30n+497, 30n+659)$ .

: **for  $k = 10$**  we have  $(p, p + 200, p + 600, p + 800)$ :  $(179, 307, 563, 691)$ .

Note that these quadruplets must have one of the following four forms:  $(30n+11, 30n+211, 30n+611, 30n+811)$ ;  $(30n+17, 30n+217, 30n+617, 30n+817)$ ;  $(30n+23, 30n+223, 30n+623, 30n+823)$ ;  $(30n+29, 30n+229, 30n+629, 30n+829)$ .

### **The generalized Brun's constant for prime quadruplets**

It is known that the Brun's constant for prime quadruplets represents the sum of the reciprocals of all prime quadruplets in the restricted sense that a prime quadruplet is  $(p, p + 2, p + 6, p + 8)$ , that is  $((1/5 + 1/7 + 1/11 + 1/13) + (1/11 + 1/13 + 1/17 + 1/19) \dots)$  and is approximately equal to 0.87.

Let's see if we can find such constants for the generalized form of this prime quadruplet, i.e. the quadruplet  $(p, p + 2 \cdot k^2, p + 6 \cdot k^2, p + 8 \cdot k^2)$ .

Let's take the quadruplet  $(p, p + 8, p + 24, p + 32)$  obtained from the general quadruplet for  $k = 2$ .

:  $(1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) \approx 0.435$ ;

:  $(1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) \approx 0.459$ ;

:  $(1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) + (1/569 + 1/577 + 1/593 + 1/601) \approx 0.466$ ;

:  $(1/5 + 1/13 + 1/29 + 1/37) + (1/29 + 1/37 + 1/53 + 1/61) + (1/149 + 1/157 + 1/173 + 1/181) + (1/569 + 1/577 + 1/593 + 1/601) + (1/719 + 1/727 + 1/743 + 1/751) \approx 0.471$ .

Finally, we conjecture that the value of *generalized Brun's constant* for prime quadruplets of the form  $(p, p + 2 \cdot k^2, p + 6 \cdot k^2, p + 8 \cdot k^2)$ , for the particular case  $k = 2$ , is not greater than 0.49 and not less than 0.48.

### 3. Conjecture on the primes of the form $(q + n)2^n + 1$ where $q$ odd prime

**Abstract.** In this paper I first conjecture that for any non-null positive integer  $n$  there exist an infinity of primes  $p$  such that the number  $q = (p - 1)/2^n - n$  is also prime and than I conjecture that for any odd prime  $q$  there exist an infinity of positive integers  $n$  such that the number  $p = (q + n)2^n + 1$  is prime.

#### Conjecture:

For any non-null positive integer  $n$  there exist an infinity of primes  $p$  such that the number  $q = (p - 1)/2^n - n$  is also prime.

#### Examples:

(for  $n = 1$ )

- : for  $p = 13$ ,  $(13 - 1)/2^1 - 1 = 5$ , prime;
- : for  $p = 17$ ,  $(17 - 1)/2^1 - 1 = 7$ , prime;
- : for  $p = 29$ ,  $(29 - 1)/2^1 - 1 = 13$ , prime;
- : for  $p = 37$ ,  $(37 - 1)/2^1 - 1 = 17$ , prime;
- : for  $p = 41$ ,  $(41 - 1)/2^1 - 1 = 19$ , prime;
- : for  $p = 61$ ,  $(61 - 1)/2^1 - 1 = 29$ , prime;
- [...]
- : for  $p = 104537$ ,  $(104537 - 1)/2^1 - 1 = 52267$ , prime;
- : for  $p = 104729$ ,  $(104729 - 1)/2^1 - 1 = 52363$ , prime.

#### Examples:

(for  $n = 2$ )

- : for  $p = 29$ ,  $(29 - 1)/2^2 - 2 = 5$ , prime;
- : for  $p = 37$ ,  $(37 - 1)/2^2 - 2 = 7$ , prime;
- : for  $p = 53$ ,  $(53 - 1)/2^2 - 2 = 11$ , prime;
- : for  $p = 61$ ,  $(61 - 1)/2^2 - 2 = 13$ , prime;
- [...]
- : for  $p = 104693$ ,  $(104693 - 1)/2^2 - 2 = 26171$ , prime.
- : for  $p = 104717$ ,  $(104717 - 1)/2^2 - 2 = 26177$ , prime.

#### Examples:

(for  $n = 3$ )

- : for  $p = 113$ ,  $(113 - 1)/2^3 - 3 = 11$ , prime;
- : for  $p = 192$ ,  $(192 - 1)/2^3 - 3 = 23$ , prime;
- : for  $p = 257$ ,  $(256 - 1)/2^3 - 3 = 29$ , prime;
- : for  $p = 353$ ,  $(353 - 1)/2^3 - 3 = 41$ , prime.

#### Examples:

(for  $n = 4$ )

- : for  $p = 113$ ,  $(113 - 1)/2^4 - 4 = 3$ , prime;
- : for  $p = 337$ ,  $(337 - 1)/2^4 - 4 = 17$ , prime;
- : for  $p = 433$ ,  $(433 - 1)/2^4 - 4 = 23$ , prime.

**Examples:**(for  $n = 5$ ): for  $p = 577$ ,  $(577 - 1)/2^5 - 5 = 13$ , prime.**Examples:**(for  $n = 6$ ): for  $p = 577$ ,  $(577 - 1)/2^6 - 6 = 3$ , prime;

[...]

: for  $p = 104513$ ,  $(104513 - 1)/2^6 - 6 = 1627$ , prime.**Conjecture:**

For any odd prime  $q$  there exist an infinity of positive integers  $n$  such that the number  $p = (q + n) \cdot 2^n + 1$  is prime.

- : for  $q = 3$ , the least  $n$  for which  $p$  is prime is  $n = 4$ , because  $(3 + 4) \cdot 2^4 + 1 = 113$ , prime;
- : for  $q = 5$ , the least  $n$  for which  $p$  is prime is  $n = 1$ , because  $(5 + 1) \cdot 2^1 + 1 = 13$ , prime;
- : for  $q = 7$ , the least  $n$  for which  $p$  is prime is  $n = 1$ , because  $(7 + 1) \cdot 2^1 + 1 = 17$ , prime;
- : for  $q = 11$ , the least  $n$  for which  $p$  is prime is  $n = 2$ , because  $(11 + 2) \cdot 2^2 + 1 = 53$ , prime;
- : for  $q = 13$ , the least  $n$  for which  $p$  is prime is  $n = 1$ , because  $(13 + 1) \cdot 2^1 + 1 = 29$ , prime;
- : for  $q = 17$ , the least  $n$  for which  $p$  is prime is  $n = 1$ , because  $(17 + 1) \cdot 2^1 + 1 = 37$ , prime;
- : for  $q = 19$ , the least  $n$  for which  $p$  is prime is  $n = 1$ , because  $(19 + 1) \cdot 2^1 + 1 = 41$ , prime;
- : for  $q = 23$ , the least  $n$  for which  $p$  is prime is  $n = 2$ , because  $(23 + 2) \cdot 2^2 + 1 = 101$ , prime;
- : for  $q = 29$ , the least  $n$  for which  $p$  is prime is  $n = 1$ , because  $(29 + 1) \cdot 2^1 + 1 = 61$ , prime;
- : for  $q = 31$ , the least  $n$  for which  $p$  is prime is  $n = 5$ , because  $(31 + 5) \cdot 2^5 + 1 = 1153$ , prime [note the interesting fact that for  $n = 4$  is obtained  $(31 + 4) \cdot 2^4 + 1 = 561$ , the first absolute Fermat pseudoprime].

Taking seven larger consecutive primes were obtained:

- : for  $q = 104693$ , the least  $n$  for which  $p$  is prime is  $n = 8$ , because  $(104693 + 8) \cdot 2^8 + 1 = 26803457$ , prime;
- : for  $q = 104701$ , the least  $n$  for which  $p$  is prime is  $n = 2$ , because  $(104701 + 2) \cdot 2^2 + 1 = 418813$ , prime;
- : for  $q = 104707$ , the least  $n$  for which  $p$  is prime is  $n = 2$ , because  $(104707 + 2) \cdot 2^2 + 1 = 418837$ , prime;
- : for  $q = 104711$ , the least  $n$  for which  $p$  is prime is  $n = 4$ , because  $(104711 + 4) \cdot 2^4 + 1 = 1675441$ , prime;
- : for  $q = 104717$ , the least  $n$  for which  $p$  is prime is  $n = 7$ , because  $(104717 + 7) \cdot 2^7 + 1 = 13404673$ , prime;
- : for  $q = 104723$ , the least  $n$  for which  $p$  is prime is  $n = 1$ , because  $(104723 + 1) \cdot 2^1 + 1 = 209449$ , prime;
- : for  $q = 104729$ , the least  $n$  for which  $p$  is prime is  $n = 8$ , because  $(104729 + 8) \cdot 2^8 + 1 = 26812673$ , prime;

Note the relative small value of  $n$  for which the first prime is found!

#### 4. Two conjectures on the numbers of the form $4p^4 - 800p^2 + 5$ where $p$ is prime

**Abstract.** In this paper I state two conjectures on the numbers of the form  $4p^4 - 800p^2 + 5$ , where  $p$  is prime, i.e. that there exist an infinity of primes of such form respectively that there exist an infinity of semiprimes  $q \cdot r$  of such form, where  $r = q + 40 \cdot n$ , where  $n$  positive integer.

##### Conjecture 1:

There exist an infinity of primes  $q$  of the form  $q = 4p^4 - 800p^2 + 5$ , where  $p$  is prime.

##### Examples:

: for  $p = 3$ ,  $q = 7529$ , prime;  
: for  $p = 7$ ,  $q = 48809$ , prime;  
: for  $p = 13$ ,  $q = 249449$ , prime;  
: for  $p = 17$ ,  $q = 565289$ , prime;  
: for  $p = 31$ ,  $q = 4462889$ , prime;  
: for  $p = 41$ ,  $q = 12647849$ , prime;  
: for  $p = 43$ ,  $q = 15154409$ , prime;  
: for  $p = 53$ ,  $q = 33809129$ , prime;  
: for  $p = 67$ ,  $q = 84195689$ , prime;  
: for  $p = 71$ ,  $q = 105679529$ , prime;  
: for  $p = 83$ ,  $q = 195344489$ , prime;  
: for  $p = 101$ ,  $q = 424402409$ , prime;  
: for  $p = 127$ ,  $q = 1053481769$ , prime;  
: for  $p = 167$ ,  $q = 3133496489$ , prime;  
: for  $p = 239$ ,  $q = 13096931369$ , prime;  
: for  $p = 251$ ,  $q = 15926904809$ , prime;  
: for  $p = 307$ ,  $q = 35606895209$ , prime;  
[...]  
: for  $p = 104723$ ,  $q = 481092181583867300969$ , prime.

##### Conjecture 2:

There exist an infinity of semiprimes  $q \cdot r$  of the form  $q \cdot r = 4p^4 - 800p^2 + 5$ , where  $p$  is prime, such that  $r = q + 40 \cdot n$ , where  $n$  positive integer.

##### Examples:

: for  $p = 19$ ,  $q \cdot r = 7 \cdot 115727$  and  $115727 = 7 + 40 \cdot 2893$ ;  
: for  $p = 29$ ,  $q \cdot r = 227 \cdot 15427$  and  $15427 = 227 + 40 \cdot 380$ ;  
: for  $p = 37$ ,  $q \cdot r = 7 \cdot 1227407$  and  $1227407 = 7 + 40 \cdot 30685$ ;  
: for  $p = 59$ ,  $q \cdot r = 73 \cdot 702113$  and  $702113 = 73 + 40 \cdot 17551$ ;  
: for  $p = 61$ ,  $q \cdot r = 7 \cdot 8337167$  and  $8337167 = 7 + 40 \cdot 208429$ ;  
: for  $p = 97$ ,  $q \cdot r = 797 \cdot 453757$  and  $453757 = 797 + 40 \cdot 11324$ ;  
: for  $p = 109$ ,  $q \cdot r = 487 \cdot 1178927$  and  $1178927 = 487 + 40 \cdot 29461$ ;

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:   for p = 113, q*r = 2203*300683 and 300683 = 2203 + 40*7462;
:   for p = 137, q*r = 433*3288953 and 3288953 = 433 + 40*82213;
:   for p = 151, q*r = 31237*67157 and 67157 = 31237 + 40*898;
:   for p = 157, q*r = 233*10515073 and 10515073 = 233 + 40*262871;
:   for p = 179, q*r = 10973*376573 and 376573 = 10973 + 40*9140;
:   for p = 181, q*r = 14783*292183 and 292183 = 14783 + 40*6935;
:   for p = 191, q*r = 7*764662607 and 764662607 = 7 + 40*19116565;
:   for p = 197, q*r = 3607*1678847 and 1678847 = 3607 + 40*41881;
:   for p = 223, q*r = 967*10270607 and 10270607 = 967 + 40*256741;
:   for p = 227, q*r = 66863*159463 and 159463 = 66863 + 40*2315;
:   for p = 229, q*r = 7*1577455247 and 1577455247 = 7 + 40*39436381;
[...]
```

## 5. Three conjectures on the numbers of the form $p(p + 4n) - 60n$ where $p$ and $p + 4n$ primes

**Abstract.** In this paper I present three conjectures on the numbers of the form  $p^*(p + 4*n) - 60*n$ , where  $p$  and  $p + 4*n$  are primes, more accurate a general conjecture and two particular ones, on the numbers of the form  $p^*(p + 4) - 60$  respectively  $p^*(p + 20) - 300$ .

### Note:

The numbers of the form  $p^*(p + 4*n) - 60*n$ , where  $p$  and  $p + 4*n$  are primes, seem to have special attributes.

### Conjecture 1:

There exist an infinity of primes of the form  $p^*(p + 4*n) - 60*n$ , where  $p$  and  $p + 4*n$  are primes, for any  $n$  non-null positive integer.

#### 1.

Let's take the positive numbers of the form  $p*q - 60$ , where  $p$  and  $q = p + 4$  are both primes:

- : for  $(p, q) = (7, 11)$  is obtained 17, prime;
- : for  $(p, q) = (13, 17)$  is obtained  $161 = 7*23$ ;
- : for  $(p, q) = (19, 23)$  is obtained  $377 = 13*29$ ;
- : for  $(p, q) = (37, 41)$  is obtained  $1457 = 31*47$ ;
- [...]
- : for  $(p, q) = (104323, 104327)$  is obtained  $73*101*1033*1429$  (we note the prime factors with  $a, b, c, d, a < b < c < d$ , and it can be seen that  $b*c - a*d = 16$ );
- : for  $(p, q) = (104239, 104243)$  is obtained  $61*1709*104233$  (it can be seen that  $a*b - c = 16$ );
- : for  $(p, q) = (104707, 104711)$  is obtained  $10963974617 = 104701*104717$  (it can be seen that  $b - a = 16$ );

### Conjecture 2:

For any composite number of the form  $p*q - 60$ , where  $p$  and  $q = p + 4$  are both primes, is true that its prime factors can be divided in two sets in such a way such that the result of the subtraction of the product of some of them (or one of them) from the product of the others (or the other one of them) is equal to 16.

#### 2.

Let's take the positive numbers of the form  $p*q - 120$ , where  $p$  and  $q = p + 8$  are both primes: the sequence of primes of this form is 83, 953, 3833, 8513, 10889, 18089 (...), obtained for  $(p, q) = (11, 19), (29, 37), (59, 67), (89, 97), (101, 109), (131, 139)$ ...

### 3.

Let's take the positive numbers of the form  $p*q - 180$ , where  $p$  and  $q = p + 12$  are both primes: the sequence of primes of this form is 73, 313, 409, 1009, 1993, 2593, 4273, 5113 (...), obtained for  $(p, q) = (11, 23), (17, 29), (19, 31), (29, 41), (41, 53), (47, 59), (61, 73), (67, 79)$ ...

### 4.

Let's take the positive numbers of the form  $p*q - 240$ , where  $p$  and  $q = p + 16$  are both primes: the sequence of primes of this form is 137, 1217, 1721, 6257 (...), obtained for  $(p, q) = (13, 29), (31, 47), (37, 53), (73, 89)$ ...

### 5.

Let's take the positive numbers of the form  $p*q - 300$ , where  $p$  and  $q = p + 20$  are both primes:

- : for  $(p, q) = (11, 31)$  is obtained 41, prime;
- : for  $(p, q) = (17, 37)$  is obtained  $329 = 7*47$ ;
- : for  $(p, q) = (23, 43)$  is obtained  $689 = 13*53$ ;
- : for  $(p, q) = (41, 61)$  is obtained  $2201 = 31*71$ ;
- [...]
- : for  $(p, q) = (104681, 104701)$  is obtained  $7*19*787*104711$  (we note the prime factors with  $a, b, c, d, a < b < c < d$  and it can be seen that  $d - a*b*c = 40$ ;
- : for  $(p, q) = (104639, 104659)$  is obtained  $7*17*47*131*14947$  (it can be seen that  $b*c*d - a*e = 40$ );
- : for  $(p, q) = (104471, 104491)$  is obtained  $7*31*3371*14923$  (it can be seen that  $b*c - a*d = 40$ );
- : for  $(p, q) = (104327, 104347)$  is obtained  $11*53*73*179*1429$  (it can be seen that  $a*b*d - c*e = 40$ );

### Conjecture 3:

For any composite number of the form  $p*q - 300$ , where  $p$  and  $q = p + 20$  are both primes, is true that its prime factors can be divided in two sets in such a way such that the result of the subtraction of the product of some of them (or one of them) from the product of the others (or the other one of them) is equal to 40.

## 6. Conjecture on an infinity of triplets of primes generated by each 3-Poulet number

**Abstract.** In this paper I present the following conjecture: for any 3-Poulet number (Fermate pseudoprime to base two with three prime factors)  $P = x*y*z$  is true that there exist an infinity of triplets of primes  $[a, b, c]$  such that  $x*a + a - x = y*b + b - y = z*c + c - z$ .

### Conjecture:

For any 3-Poulet number (Fermate pseudoprime to base two with three prime factors)  $P = x*y*z$  is true that there exist an infinity of triplets of primes  $[a, b, c]$  such that  $x*a + a - x = y*b + b - y = z*c + c - z$ .

The sequence of 3-Poulet numbers is: 561, 645, 1105, 1729, 1905, 2465, 2821, 4371, 6601, 8481, 8911, 10585, 12801, 13741, 13981, 15841 (...). See the sequence A215672 that I posted on OEIS.

### Examples:

For  $P = 561 = 3*11*17$ ,

we need to find  $[a, b, c]$  such that  $4*a - 3 = 12*b - 11 = 18*c - 17$ ; for this,  $[a, b, c]$  must be of the form  $[9*n + 1, 3*n + 1, 2*n + 1]$ , where  $n$  can't be odd, can't be of the form  $3*k + 1$  and also can't have the last digit 2, 6 or 8. The least  $n$  for which  $[a, b, c]$  are all three primes is  $n = 20$  which gives us  $[a, b, c] = [181, 61, 41]$ . The following such triplet is  $[a, b, c] = [487, 163, 109]$  corresponding to  $n = 54$ .

For  $P = 645 = 3*5*43$ ,

we need to find  $[a, b, c]$  such that  $4*a - 3 = 6*b - 5 = 44*c - 43$ ; for this,  $[a, b, c]$  must be of the form  $[33*n + 1, 22*n + 1, 3*n + 1]$ , where  $n$  can't be odd, can't be of the form  $3*k + 2$  and also can't have the last digit 2 or 8. The least  $n$  for which  $[a, b, c]$  are all three primes is  $n = 4$  which gives us  $[a, b, c] = [133, 89, 13]$ . The following such triplet is  $[a, b, c] = [199, 133, 19]$  corresponding to  $n = 6$ .

For  $P = 1105 = 5*13*17$ ,

we need to find  $[a, b, c]$  such that  $6*a - 5 = 14*b - 13 = 18*c - 17$ ; for this,  $[a, b, c]$  must be of the form  $[21*n + 1, 9*n + 1, 7*n + 1]$ , where  $n$  can't be odd, can't be of the form  $3*k + 2$  and also can't have the last digit 2, 4 or 6. The least  $n$  for which  $[a, b, c]$  are all three primes is  $n = 18$  which gives us  $[a, b, c] = [379, 163, 127]$ . The following such triplet is  $[a, b, c] = [631, 271, 211]$  corresponding to  $n = 30$ .

For  $P = 1729 = 7*13*19$ , we need to find  $[a, b, c]$  such that  $8*a - 7 = 14*b - 13 = 20*c - 19$ ; for this,  $[a, b, c]$  must be of the form  $[35*n + 1, 20*n + 1, 14*n + 1]$ , where  $n$  can't be odd, can't be of the form  $3*k + 1$  and also can't have the last digit 6. The least  $n$  for which  $[a, b, c]$  are all three primes is  $n = 2$  which gives us  $[a, b, c] = [71, 41, 29]$ . The following such triplet is  $[a, b, c] = [491, 281, 197]$  corresponding to  $n = 14$ .

For  $P = 1905 = 3 \cdot 5 \cdot 127$ , we need to find  $[a, b, c]$  such that  $4 \cdot a - 3 = 6 \cdot b - 5 = 128 \cdot c - 127$ ; for this,  $[a, b, c]$  must be of the form  $[96 \cdot n + 1, 64 \cdot n + 1, 3 \cdot n + 1]$ , where  $n$  can't be odd, can't be of the form  $3 \cdot k + 2$  and also can't have the last digit 4, 6 or 8. The least  $n$  for which  $[a, b, c]$  are all three primes is  $n = 12$  which gives us  $[a, b, c] = [1153, 769, 37]$ . The following such triplet is  $[a, b, c] = [2113, 1409, 67]$  corresponding to  $n = 22$ .

For  $P = 2465 = 5 \cdot 17 \cdot 29$ , we need to find  $[a, b, c]$  such that  $6 \cdot a - 5 = 18 \cdot b - 17 = 30 \cdot c - 29$ ; for this,  $[a, b, c]$  must be of the form  $[15 \cdot n + 1, 5 \cdot n + 1, 3 \cdot n + 1]$ , where  $n$  can't be odd, can't be of the form  $3 \cdot k + 1$  and also can't have the last digit 8. The least  $n$  for which  $[a, b, c]$  are all three primes is  $n = 2$  which gives us  $[a, b, c] = [31, 11, 7]$ . The following such triplet is  $[a, b, c] = [181, 61, 37]$  corresponding to  $n = 12$ .

For  $P = 2821 = 7 \cdot 13 \cdot 31$ , we need to find  $[a, b, c]$  such that  $8 \cdot a - 7 = 14 \cdot b - 13 = 32 \cdot c - 31$ ; for this,  $[a, b, c]$  must be of the form  $[28 \cdot n + 1, 16 \cdot n + 1, 7 \cdot n + 1]$ , where  $n$  can't be odd, can't be of the form  $3 \cdot k + 2$  and also can't have the last digit 2, 4 or 8. The least  $n$  for which  $[a, b, c]$  are all three primes is  $n = 16$  which gives us  $[a, b, c] = [449, 257, 113]$ . The following such triplet is  $[a, b, c] = [841, 481, 211]$  corresponding to  $n = 30$ .

## 7. Observation on the numbers $4p^2 - 2p - 1$ where $p$ and $2p - 1$ are primes

**Abstract.** In this paper I observe that many numbers of the form  $4p^2 - 2p - 1$ , where  $p$  and  $2p - 1$  are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to  $d*Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = n*d - n + 1$ ; (iii) they are equal to  $d*Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = n*d + n - 1$ , and I make few related notes.

### Observation:

Many numbers of the form  $N = 4p^2 - 2p - 1$ , where  $p$  and  $2p - 1$  are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to  $d*Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = n*d - n + 1$ ; (iii) they are equal to  $d*Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = n*d + n - 1$ .

### Verifying the observation:

(true for the first 27 odd primes  $p$  for which  $2p - 1$  is also prime)

Note that if  $p$  is prime of the form  $10k + 9$  than the least prime factor of  $N$  is 5 and obviously then  $N$  respects the condition (ii) or (iii).

Also note that if  $d$  is equal to 11 and  $Q$  is of the form  $10k + 1$  is obviously respected condition (ii).

: for  $p = 3$ ,  $N = 29$ , prime;  
: for  $p = 7$ ,  $N = 181$ , prime;  
: for  $p = 19$ ,  $N$  divisible by 5;  
: for  $p = 31$ ,  $N = 19*199$  and  $199 = 11*19 - 10$ ;  
: for  $p = 37$ ,  $N = 11*491$ ,  $d = 11$  and  $Q = 10k + 1$ ;  
: for  $p = 79$ ,  $N$  divisible by 5;  
: for  $p = 97$ ,  $N = 37441$ , prime;  
: for  $p = 139$ ,  $N$  divisible by 5;  
: for  $p = 157$ ,  $N = 29*3389$  and  $3389 = 121*29 - 120$ ;  
: for  $p = 199$ ,  $N$  divisible by 5;  
: for  $p = 211$ ,  $N = 11*16151$ ,  $d = 11$  and  $Q = 10k + 1$ ;  
: for  $p = 229$ ,  $N$  divisible by 5;  
: for  $p = 271$ ,  $N = 293221$ , prime;  
: for  $p = 307$ ,  $N = 89*4229$  and  $4229 = 47*89 + 46$ ;  
: for  $p = 331$ ,  $N = 29*15089$  and  $15089 = 503*29 + 502$ ;  
: for  $p = 337$ ,  $N = 453601$ , prime;  
: for  $p = 367$ ,  $N = 11*48911$ ,  $d = 11$  and  $Q = 10k + 1$ ;  
: for  $p = 379$ ,  $N$  divisible by 5;  
: for  $p = 439$ ,  $N$  divisible by 5;  
: for  $p = 499$ ,  $N$  divisible by 5;  
: for  $p = 547$ ,  $N = 1195741$ , prime;  
: for  $p = 577$ ,  $N = 241*5521$  and  $5521 = 23*241 - 22$ ;  
: for  $p = 601$ ,  $N = 19*75979$  and  $75979 = 4221*19 - 4220$ ;

- : for  $p = 607$ ,  $N = 11 \cdot 133871$ ,  $d = 11$  and  $Q = 10k + 1$ ;
- : for  $p = 619$ ,  $N$  divisible by 5;
- : for  $p = 661$ ,  $N = 131 \cdot 13331$  and  $13331 = 101 \cdot 131 + 100$ .

**Notes:**

: Some numbers of this form meet another condition, i.e. they are equal to  $d \cdot Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = (n \cdot d - n + m)/m$ , or respectively  $Q = (n \cdot d + n - m)/m$ . An example: for  $p = 691$ ,  $N = 149 \cdot 12809$  and  $12809 = (427 \cdot 149 + 427 - 5)/5$ ;

: Some numbers of this form meet yet another condition, i.e. they are equal to  $d \cdot Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and the number  $Q - d + 1$  is prime or respectively the number  $Q + d - 1$  is prime. An example: for  $p = 727$ ,  $N = 139 \cdot 15199$  and  $15199 - 139 + 1 = 15061$ , prime.

## 8. Observation on the numbers $4p^2 + 2p + 1$ where $p$ and $2p - 1$ are primes

**Abstract.** In this paper I observe that many numbers of the form  $4p^2 + 2p + 1$ , where  $p$  and  $2p - 1$  are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to  $d*Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = (n*d - n + m)/m$ ; (iii) they are equal to  $d*Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = (n*d + n - m)/m$ , and I make few related notes.

### Observation:

Many numbers of the form  $4p^2 + 2p + 1$ , where  $p$  and  $2p - 1$  are odd primes, meet one of the following three conditions: (i) they are primes; (ii) they are equal to  $d*Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = (n*d - n + m)/m$ ; (iii) they are equal to  $d*Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and  $Q = (n*d + n - m)/m$ .

### Verifying the observation:

(true for the first 27 odd primes  $p$  for which  $2p - 1$  is also prime)

Note that if  $d$  is equal to 7 is obviously respected condition (i) or condition (ii).

- : for  $p = 3$ ,  $N = 43$ , prime;
- : for  $p = 7$ ,  $N = 211$ , prime;
- : for  $p = 19$ ,  $N = 1483$ , prime;
- : for  $p = 31$ ,  $N = 3907$ , prime;
- : for  $p = 37$ ,  $N = 7*13*61$  so  $d = 7$ ;
- : for  $p = 79$ ,  $N = 7*37*97$  so  $d = 7$ ;
- : for  $p = 97$ ,  $N = 37831$ , prime;
- : for  $p = 139$ ,  $N = 77563$ , prime;
- : for  $p = 157$ ,  $N = 98911$ , prime;
- : for  $p = 199$ ,  $N = 158803$ , prime;
- : for  $p = 211$ ,  $N = 7*7*3643$  so  $d = 7$ ;
- : for  $p = 229$ ,  $N = 13*16171$  and  $16171 = (2695*13 - 2695 + 2)/2$ ;
- : for  $p = 271$ ,  $N = 13*22639$  and  $22639 = (3773*13 - 3773 + 2)/2$ ;
- : for  $p = 307$ ,  $N = 13*29047$  and  $29047 = (4841*13 - 4841 + 2)/2$ ;
- : for  $p = 331$ ,  $N = 7*62701$  so  $d = 7$ ;
- : for  $p = 337$ ,  $N = 7*64993$  so  $d = 7$ ;
- : for  $p = 367$ ,  $N = 79*6829$  and  $6829 = (683*79 + 683 - 8)/8$ ;
- : for  $p = 379$ ,  $N = 7*82189$  so  $d = 7$ ;
- : for  $p = 439$ ,  $N = 771763$ , prime;
- : for  $p = 499$ ,  $N = 7*7*20347$  so  $d = 7$ ;
- : for  $p = 547$ ,  $N = 7*171133$  so  $d = 7$ ;
- : for  $p = 577$ ,  $N = 43*30997$  and  $30997 = (738*43 - 738 + 1)/1$ ;
- : for  $p = 601$ ,  $N = 1446007$ , prime;
- : for  $p = 607$ ,  $N = 31*47581$  and  $47581 = (1586*31 - 1586 + 1)/1$ ;
- : for  $p = 619$ ,  $N = 13*117991$  and  $117991 = (19665*13 - 19665 + 2)/2$ ;
- : for  $p = 661$ ,  $N = 13*134539$  and  $134539 = (22423*13 - 22423 + 2)/2$ .

**Note:**

: Some numbers of this form meet another condition, i.e. they are equal to  $d \cdot Q$ , where  $d$  is the least prime factor and  $Q$  the product of the others, and the number  $Q - d + 1$  is prime or respectively the number  $Q + d - 1$  is prime. An example: for  $p = 691$ ,  $N = 43 \cdot 44449$  and  $44449 + 43 - 1 = 44491$ , prime.

## 9. Conjecture on the numbers $6pq + 1$ where $p$ and $q$ primes and $q = kp - k + 1$

**Abstract.** In this paper I make the following conjecture on the numbers of the form  $n = 6pq + 1$ , where  $p$  and  $q$  are primes and  $q = kp - k + 1$ : There exist an infinity of  $n$  primes for any  $k$  positive integer,  $k > 1$ . Note that the conjecture implies that there exist an infinity of pairs of primes  $[p, q]$  such that  $q = kp - k + 1$ , for any  $k$  positive integer,  $k > 1$ , which I already conjectured in previous papers, as well as that there exist an infinity of pairs of primes  $[p, q]$  such that  $q = kp + k - 1$ , for any  $k$  positive integer,  $k > 1$ .

### Conjecture:

There exist an infinity of primes  $n$  of the form  $n = 6pq + 1$ , where  $p$  and  $q$  are primes and  $q = kp - k + 1$ , for any  $k$  positive integer,  $k > 1$ .

Note that the conjecture implies that there exist an infinity of pairs of primes  $[p, q]$  such that  $q = kp - k + 1$ , for any  $k$  positive integer,  $k > 1$ , which I already conjectured in previous papers, as well as that there exist an infinity of pairs of primes  $[p, q]$  such that  $q = kp + k - 1$ , for any  $k$  positive integer,  $k > 1$ .

The sequence of these primes for  $k = 2$  ( $q = 2p - 1$ ):

: 547, 4219, 74419, 112327, 627919, 879667, 2310019 (...), obtained for  $[p, q] = [7, 13], [19, 37], [79, 157], [97, 193], [229, 457], [271, 541], [439, 877]$ ...

See A005382 in OEIS for primes  $p$  such that  $2p - 1$  also prime.

The sequence of these primes for  $k = 3$  ( $q = 3p - 2$ ):

: 2887, 39199, 49927, 79999, 336199, 587527, 3338527 (...), obtained for  $[p, q] = [13, 37], [47, 139], [53, 157], [67, 199], [137, 409], [181, 541], [431, 1291]$ ...

See A088878 in OEIS for primes  $p$  such that  $3p - 2$  also prime.

The sequence of these primes for  $k = 4$  ( $q = 4p - 3$ ):

: 2707, 82483, 283183, 530143, 872107, 1655323 (...), obtained for  $[p, q] = [11, 41], [59, 233], [109, 433], [149, 593], [191, 761], [263, 1049]$ ...

See A157978 in OEIS for primes  $p$  such that  $4p - 3$  also prime.

## 10. Three conjectures on the numbers $6pq + 1$ where $p$ and $q$ primes and $q = 2p - 1$

**Abstract.** In this paper I make the following three conjectures on the numbers of the form  $n = 6pq + 1$ , where  $p$  and  $q$  are primes and  $q = 2p - 1$ : (I) There exist an infinity of  $n$  primes; (II) There exist an infinity of  $n$  semiprimes; (III) There exist an infinity of  $n$  composites with three or more prime factors, 7 being one of them. Note that for all the first 46 pairs of primes  $[p, q]$  with the property mentioned (see the sequence A005382 in OEIS for these primes) the number  $n$  obtained belongs to one of the three sequences considered by the three conjectures above.

### Conjecture I:

There exist an infinity of primes  $n$  of the form  $n = 6pq + 1$ , where  $p$  and  $q$  are primes and  $q = 2p - 1$ .

The sequence of these primes is:

: 547 ( $= 6 \cdot 7 \cdot 13 + 1$ ), 4219 ( $= 6 \cdot 19 \cdot 37 + 1$ ), 74419 ( $= 6 \cdot 79 \cdot 157 + 1$ ), 112327 ( $= 6 \cdot 97 \cdot 193 + 1$ ), 627919 ( $= 6 \cdot 229 \cdot 457 + 1$ ), 879667 ( $= 6 \cdot 271 \cdot 541 + 1$ ), 2310019 ( $= 6 \cdot 439 \cdot 877 + 1$ ), 5725627 ( $= 6 \cdot 691 \cdot 1381 + 1$ ), 6337987 ( $= 6 \cdot 727 \cdot 1453 + 1$ ), 16447867 ( $= 6 \cdot 1171 \cdot 2341 + 1$ ), 23478019 ( $= 6 \cdot 1399 \cdot 2797 + 1$ ), 32937847 ( $= 6 \cdot 1657 \cdot 3313 + 1$ )...

### Conjecture II:

There exist an infinity of semiprimes  $n$  of the form  $n = 6pq + 1$ , where  $p$  and  $q$  are primes and  $q = 2p - 1$ .

The sequence of these semiprimes is:

: 11347 ( $= 7 \cdot 1621 = 6 \cdot 31 \cdot 61 + 1$ ), 16207 ( $= 19 \cdot 853 = 6 \cdot 37 \cdot 73 + 1$ ), 1129147 ( $= 79 \cdot 14293 = 6 \cdot 307 \cdot 613 + 1$ ), 1312747 ( $= 43 \cdot 30529 = 6 \cdot 331 \cdot 661 + 1$ ), 2985019 ( $= 163 \cdot 18313 = 6 \cdot 499 \cdot 997 + 1$ ), 4330807 ( $= 13 \cdot 333139 = 6 \cdot 601 \cdot 1201 + 1$ ), 4417747 ( $= 19 \cdot 232513 = 6 \cdot 607 \cdot 1213 + 1$ ), 5239087 ( $= 7 \cdot 748441 = 6 \cdot 661 \cdot 1321$ ), 7887787 ( $= 151 \cdot 52237 = 6 \cdot 811 \cdot 1621 + 1$ ), 9224287 ( $= 211 \cdot 43717 = 6 \cdot 877 \cdot 1753 + 1$ ), 10530007 ( $= 1279 \cdot 8233 = 6 \cdot 937 \cdot 1873 + 1$ ), 13706719 ( $= 13 \cdot 1054363 = 6 \cdot 1069 \cdot 2137 + 1$ ), 18354607 ( $= 1153 \cdot 15919 = 6 \cdot 1237 \cdot 2473 + 1$ ), 19622419 ( $= 61 \cdot 19622419 = 6 \cdot 1279 \cdot 2557 + 1$ ), 20178727 ( $= 37 \cdot 545371 = 6 \cdot 1297 \cdot 2593 + 1$ ), 24495919 ( $= 7 \cdot 3499417 = 6 \cdot 1429 \cdot 2857 + 1$ ), 28118347 ( $= 19 \cdot 1479913 = 6 \cdot 1531 \cdot 3061 + 1$ ), 31056919 ( $= 1993 \cdot 15583 = 6 \cdot 1609 \cdot 3217 + 1$ )...

### Conjecture III:

There exist an infinity of  $n$  composites with three or more prime factors, 7 being one of them, of the form  $n = 6pq + 1$ , where  $p$  and  $q$  are primes and  $q = 2p - 1$ .

The sequence of these numbers is:

: 294847 ( $= 7*73*577 = 6*157*313 + 1$ ), 474019 ( $= 7*13*5209 = 6*199*397 + 1$ ),  
532987 ( $= 7*13*5857 = 6*211*421 + 1$ ), 1360807 ( $= 7*31*6271 = 6*337*673 + 1$ ),  
1614067 ( $= 7*13*17737 = 6*367*733 + 1$ ), 1721419 ( $= 7^2*19*43^2 = 6*379*757 + 1$ ),  
3587227 ( $= 7*31*61*271 = 6*547*1093 + 1$ ), 3991687 ( $= 7^2*81463 = 6*577*1153 + 1$ ),  
4594219 ( $= 7*19*34543 = 6*619*1237 + 1$ ), 8241919 ( $= 7*73*127^2 = 6*829*1657 + 1$ ),  
11215267 ( $= 7^2*228883 = 6*967*1933 + 1$ ), 11922127 ( $= 7*79*21559 = 6*997*1993 + 1$ ),  
12210919 ( $= 7*61*28597 = 6*1009*2017 + 1$ ), 31755787 ( $= 7*433*10477 = 6*1627*3253 + 1$ )...

**Note:**

For all the first 46 pairs of primes  $[p, q]$  with the property mentioned (see the sequence A005382 in OEIS for these primes) the number  $n$  obtained belongs to one of the three sequences considered by the three conjectures above.

# **11. Any square of a prime larger than 7 can be written as $30n^2 + 60n + p$ where p prime or power of prime**

**Abstract.** In this paper I make the following conjecture: Any square of a prime larger than 7 can be written as  $30n^2 + 60n + p$ , where p prime or power of prime and n positive integer.

## **Conjecture:**

Any square of a prime larger than 7 can be written as  $30n^2 + 60n + p$ , where p prime or power of prime and n positive integer.

## **Verifying the conjecture:**

(for the first fifteen primes larger than 7)

:  $11^2 = 121 = 30 \cdot 1^2 + 60 \cdot 1 + 31$ ;  
:  $13^2 = 169 = 30 \cdot 1^2 + 60 \cdot 1 + 79$ ;  
:  $17^2 = 289 = 30 \cdot 1^2 + 60 \cdot 1 + 199 = 30 \cdot 2^2 + 60 \cdot 2 + 7^2$ ;  
:  $19^2 = 361 = 30 \cdot 1^2 + 60 \cdot 1 + 271 = 30 \cdot 2^2 + 60 \cdot 2 + 11^2$ ;  
:  $23^2 = 529 = 30 \cdot 1^2 + 60 \cdot 1 + 439 = 30 \cdot 2^2 + 60 \cdot 2 + 17^2 = 30 \cdot 3^2 + 60 \cdot 3 + 79$ ;  
:  $29^2 = 841 = 30 \cdot 1^2 + 60 \cdot 1 + 751 = 30 \cdot 2^2 + 60 \cdot 2 + 601 = 30 \cdot 4^2 + 60 \cdot 4 + 11^2$ ;  
:  $31^2 = 961 = 30 \cdot 4^2 + 60 \cdot 4 + 241$ ;  
:  $37^2 = 1369 = 30 \cdot 1^2 + 60 \cdot 1 + 1279 = 30 \cdot 2^2 + 60 \cdot 2 + 1129 = 30 \cdot 3^2 + 60 \cdot 3 + 919$ ;  
:  $41^2 = 1681 = 30 \cdot 3^2 + 60 \cdot 3 + 1231 = 30 \cdot 4^2 + 60 \cdot 4 + 31^2 = 30 \cdot 5^2 + 60 \cdot 5 + 631 = 30 \cdot 6^2 + 60 \cdot 6 + 241$ ;  
:  $43^2 = 1849 = 30 \cdot 1^2 + 60 \cdot 1 + 1759 = 30 \cdot 2^2 + 60 \cdot 2 + 1609 = 30 \cdot 3^2 + 60 \cdot 3 + 1399 = 30 \cdot 4^2 + 60 \cdot 4 + 1129 = 30 \cdot 6^2 + 60 \cdot 6 + 409$ ;  
:  $47^2 = 2209 = 30 \cdot 3^2 + 60 \cdot 3 + 1759 = 30 \cdot 4^2 + 60 \cdot 4 + 1489 = 30 \cdot 6^2 + 60 \cdot 6 + 769$ ;  
:  $53^2 = 2809 = 30 \cdot 1^2 + 60 \cdot 1 + 2719 = 30 \cdot 4^2 + 60 \cdot 4 + 2089 = 30 \cdot 5^2 + 60 \cdot 5 + 1759 = 30 \cdot 6^2 + 60 \cdot 6 + 37^2 = 30 \cdot 7^2 + 60 \cdot 7 + 919 = 30 \cdot 8^2 + 60 \cdot 8 + 409$ ;  
:  $59^2 = 3481 = 30 \cdot 1^2 + 60 \cdot 1 + 3391$ ;  
:  $61^2 = 3721 = 30 \cdot 1^2 + 60 \cdot 1 + 3631 = 30 \cdot 2^2 + 60 \cdot 2 + 59^2 = 30 \cdot 3^2 + 60 \cdot 3 + 3271 = 30 \cdot 4^2 + 60 \cdot 4 + 3001 = 30 \cdot 5^2 + 60 \cdot 5 + 2671 = 30 \cdot 6^2 + 60 \cdot 6 + 2281 = 30 \cdot 7^2 + 60 \cdot 7 + 1831 = 30 \cdot 8^2 + 60 \cdot 8 + 1321 = 30 \cdot 9^2 + 60 \cdot 9 + 751 = 30 \cdot 10^2 + 60 \cdot 10 + 11^2$ ;  
:  $67^2 = 4489 = 30 \cdot 4^2 + 60 \cdot 4 + 3769 = 30 \cdot 6^2 + 60 \cdot 6 + 3049 = 30 \cdot 8^2 + 60 \cdot 8 + 2089 = 30 \cdot 11^2 + 60 \cdot 11 + 199$ .

## 12. Any square of a prime larger than 11 can be written as $60n^2 + 90n + p$ where p prime or power of prime

**Abstract.** In this paper I make the following conjecture: Any square of a prime larger than 11 can be written as  $60n^2 + 90n + p$ , where p prime or power of prime and n positive integer.

### Conjecture:

Any square of a prime larger than 11 can be written as  $60n^2 + 90n + p$ , where p prime or power of prime and n positive integer.

### Verifying the conjecture:

(for the first fifteen primes larger than 11)

$$\begin{aligned}
 : & 13^2 = 169 = 60 \cdot 1^2 + 90 \cdot 1 + 19; \\
 : & 17^2 = 289 = 60 \cdot 1^2 + 90 \cdot 1 + 139; \\
 : & 19^2 = 361 = 60 \cdot 1^2 + 90 \cdot 1 + 211; \\
 : & 23^2 = 529 = 60 \cdot 1^2 + 90 \cdot 1 + 379 = 60 \cdot 2^2 + 90 \cdot 2 + 109; \\
 : & 29^2 = 841 = 60 \cdot 1^2 + 90 \cdot 1 + 691 = 60 \cdot 2^2 + 90 \cdot 2 + 421 = 60 \cdot 3^2 + 90 \cdot 3 + 31; \\
 : & 31^2 = 961 = 60 \cdot 1^2 + 90 \cdot 1 + 811 = 60 \cdot 2^2 + 90 \cdot 2 + 541 = 60 \cdot 3^2 + 90 \cdot 3 + 151; \\
 : & 37^2 = 1369 = 60 \cdot 4^2 + 90 \cdot 4 + 7^2; \\
 : & 41^2 = 1681 = 60 \cdot 1^2 + 90 \cdot 1 + 1531 = 60 \cdot 4^2 + 90 \cdot 4 + 19^2; \\
 : & 43^2 = 1849 = 60 \cdot 1^2 + 90 \cdot 1 + 1699 = 60 \cdot 2^2 + 90 \cdot 2 + 1429 = 60 \cdot 3^2 + 90 \cdot 3 + 1039 \\
 & = 60 \cdot 4^2 + 90 \cdot 4 + 23^2; \\
 : & 47^2 = 2209 = 60 \cdot 2^2 + 90 \cdot 2 + 1789 = 60 \cdot 3^2 + 90 \cdot 3 + 1399; \\
 : & 53^2 = 2809 = 60 \cdot 1^2 + 90 \cdot 1 + 2659 = 60 \cdot 2^2 + 90 \cdot 2 + 2389 = 60 \cdot 3^2 + 90 \cdot 3 + 1999 \\
 & = 60 \cdot 4^2 + 90 \cdot 4 + 1489 = 60 \cdot 5^2 + 90 \cdot 5 + 859 = 60 \cdot 6^2 + 90 \cdot 6 + 109; \\
 : & 59^2 = 3481 = 60 \cdot 1^2 + 90 \cdot 1 + 3331 = 60 \cdot 2^2 + 90 \cdot 2 + 3061 = 60 \cdot 3^2 + 90 \cdot 3 + 2671 \\
 & = 60 \cdot 4^2 + 90 \cdot 4 + 2161 = 60 \cdot 5^2 + 90 \cdot 5 + 1531; \\
 : & 61^2 = 3721 = 60 \cdot 1^2 + 90 \cdot 1 + 3571 = 60 \cdot 2^2 + 90 \cdot 2 + 3301 = 60 \cdot 4^2 + 90 \cdot 4 + 7^4 = \\
 & 60 \cdot 6^2 + 90 \cdot 6 + 1021 = 60 \cdot 7^2 + 90 \cdot 7 + 151; \\
 : & 67^2 = 4489 = 60 \cdot 1^2 + 90 \cdot 1 + 4339 = 60 \cdot 4^2 + 90 \cdot 4 + 3169 = 60 \cdot 5^2 + 90 \cdot 5 + 2539 \\
 & = 60 \cdot 6^2 + 90 \cdot 6 + 1789 + 60 \cdot 7^2 + 90 \cdot 7 + 919; \\
 : & 71^2 = 5041 = 60 \cdot 2^2 + 90 \cdot 2 + 4621 = 60 \cdot 3^2 + 90 \cdot 3 + 4231 = 60 \cdot 4^2 + 90 \cdot 4 + 61^2 \\
 & = 60 \cdot 6^2 + 90 \cdot 6 + 2341 + 60 \cdot 7^2 + 90 \cdot 7 + 1471.
 \end{aligned}$$

### 13. On the numbers of the form $pq+10^k$ where $p$ and $q$ are emirps

**Abstract.** In this paper I make the following observation: there are many primes among the numbers of the form  $p*q + 10^k$ , where  $p$  and  $q$  are emirps (reversible primes but different one from the other) and  $k$  is a positive integer; to highlight the observation I will search the least  $k$  for which the number  $p*q + 10^k$  is prime, for few pairs of emirps  $[p, q]$ .

#### Observation:

There are many primes among the numbers of the form  $p*q + 10^k$ , where  $p$  and  $q$  are emirps (reversible primes but different one from the other) and  $k$  is a positive integer. To highlight the observation I will search the least  $k$  for which the number  $p*q + 10^k$  is prime, for few pairs of emirps  $[p, q]$ . Of course, if there are many low values of  $k$ , the observation is verified.

The sequence of emirps:

13, 17, 31, 37, 71, 73, 79, 97, 107, 113, 149, 157, 167, 179, 199, 311, 337, 347, 359, 389, 701, 709, 733, 739, 743, 751, 761, 769, 907, 937, 941, 953, 967, 971, 983, 991, 1009, 1021, 1031, 1033, 1061, 1069, 1091, 1097, 1103, 1109, 1151, 1153, 1181, 1193, 1201, 1213, 1217, 1223, 1229, 1231, 1237, 1249, 1259, 1279, 1283 (...)

(for more terms see A006567 in OEIS)

:  $13*31 + 100 = 503$ , prime, so the least  $k$  is 2;  
:  $17*71 + 100 = 1307$ , prime, so the least  $k$  is 2;  
:  $37*73 + 10 = 2711$ , prime, so the least  $k$  is 1;  
:  $79*97 + 10 = 7673$ , prime, so the least  $k$  is 1;  
:  $107*701 + 10 = 75017$ , prime, so the least  $k$  is 1;  
:  $113*311 + 100 = 35153$ , prime, so the least  $k$  is 1;  
:  $149*941 + 1000 = 141209$ , prime, so the least  $k$  is 3;  
:  $157*751 + 10 = 117917$ , prime, so the least  $k$  is 1;  
:  $167*761 + 10000 = 137087$ , prime, so the least  $k$  is 4;  
:  $179*971 + 10 = 173819$ , prime, so the least  $k$  is 1;  
:  $337*733 + 10 = 247031$ , prime, so the least  $k$  is 1;  
:  $347*743 + 100 = 257921$ , prime, so the least  $k$  is 2;  
:  $359*953 + 1000 = 343127$ , prime, so the least  $k$  is 3;  
:  $389*983 + 100000 = 482387$ , prime, so the least  $k$  is 5;  
:  $709*907 + 10 = 643073$ , prime, so the least  $k$  is 1;  
:  $739*937 + 10 = 692453$ , prime, so the least  $k$  is 1;  
:  $1009*9001 + 100000000 = 109082009$ , prime, so the least  $k$  is 8;  
:  $1021*1201 + 100 = 1226321$ , prime, so the least  $k$  is 2;  
:  $1031*1301 + 100000 = 1441331$ , prime, so the least  $k$  is 5;  
:  $1033*3301 + 1000 = 3410933$ , prime, so the least  $k$  is 3;  
:  $1061*1601 + 1000000000 = 1001698661$ , prime, so the least  $k$  is 9;  
:  $1069*9601 + 10 = 10263479$ , prime, so the least  $k$  is 1;  
:  $1091*1901 + 100 = 2074091$ , prime, so the least  $k$  is 2;  
:  $1097*7901 + 100 = 8667497$ , prime, so the least  $k$  is 2;

:  $1103*3011 + 100 = 3321233$ , prime, so the least  $k$  is 2;  
 :  $1109*9011 + 10 = 9993209$ , prime, so the least  $k$  is 1;  
 :  $1153*3511 + 100 = 4048283$ , prime, so the least  $k$  is 2;  
 :  $1181*1811 + 10000 = 2148791$ , prime, so the least  $k$  is 4;  
 :  $1193*3911 + 10 = 4665833$ , prime, so the least  $k$  is 1;  
 :  $1213*3121 + 10000 = 3795773$ , prime, so the least  $k$  is 4;  
 :  $1217*7121 + 1000 = 8667257$ , prime, so the least  $k$  is 3;  
 :  $1229*9221 + 100 = 11332709$ , prime, so the least  $k$  is 2;  
 :  $1237*7321 + 10000000 = 19056077$ , prime, so the least  $k$  is 7;  
 :  $1249*9421 + 100 = 11766929$ , prime, so the least  $k$  is 2;  
 :  $1259*9521 + 10000 = 11996939$ , prime, so the least  $k$  is 4;  
 :  $1279*9721 + 1000000 = 13433159$ , prime, so the least  $k$  is 6;  
 :  $1283*3821 + 10 = 4902353$ , prime, so the least  $k$  is 1.

#### 14. Formula that generates a large amount of big primes and semiprimes *i.e.* $529 + 60 \cdot 10^k$

**Abstract.** In this paper I make the following observation: the formula  $529 + 60 \cdot 10^k$ , where  $k$  positive integer, seems to generate a large amount of big primes and semiprimes. Indeed, up to  $k = 32$ , this formula generates 11 primes and 11 semiprimes!

##### Observation:

The formula  $529 + 60 \cdot 10^k$ , where  $k$  positive integer, seems to generate a large amount of big primes and semiprimes. Indeed, up to  $k = 35$ , this formula generates 11 primes and 12 semiprimes!

The following terms are semiprimes:

: 589;  
: 60529;  
: 60000000529;  
: 60000000000529;  
: 600000000000000529;  
: 600000000000000000529;  
: 60000000000000000000529;  
: 6000000000000000000000529;  
: 600000000000000000000000529;  
: 60000000000000000000000000529;  
: 6000000000000000000000000000529;  
: 600000000000000000000000000000529.

The following terms are primes:

: 1129;  
: 6529;  
: 600529;  
: 6000529;  
: 600000000529;  
: 6000000000529;  
: 6000000000000529;  
: 600000000000000529;  
: 60000000000000000529;  
: 6000000000000000000529;  
: 600000000000000000000529;  
: 60000000000000000000000529.

##### Note:

This special property of the square of the prime number 23 is not shared by the other squares of primes; for instance, the formula  $p^2 + 60 \cdot 10^k$  generates, up to  $k = 35$ , only 3 primes for  $p = 7$  and only 4 primes for  $p = 11$ .

## 15. A sequence of numbers created concatenating the digit 1 twice with a prime of the form $6k - 1$

**Abstract.** In this paper I show an interesting sequence of numbers created concatenating to the right the digit 1, twice, with a prime of the form  $6k - 1$  (example of such numbers, terms of this sequence: 12929 and 15353), sequence that has, from the first 50 terms, 21 terms that are primes and 22 that are semiprimes.

### Observation:

The sequence created concatenating to the right the digit 1, twice, with a prime of the form  $6k - 1$  (example of such numbers, terms of this sequence: 12929 and 15353) seems to be particularly interesting; beside the fact that the sequence contains a lot of terms that are primes, many of the composite terms also share a special property: up to the 50-th term of the sequence, all the composites are semiprimes  $p*q$ , in which case many of these have the property that  $p + q - 1$  is a prime, or squarefree composites with three prime factors  $p*q*r$ , in which case many of these have the property that  $p*q + r - 1$  is a prime.

### The sequence of primes:

: 11717, 12323, 14747, 15959, 1107107, 1131131, 1137137, 1167167, 1173173, 1179179, 1191191, 1197197, 1239239, 1263263, 1281281, 1311311, 1317317, 1401401, 1479479, 1503503, 1509509 (...)

### The sequence of semiprimes:

: 11111, 12929, 14141, 15353, 18383, 18989, 1113113, 1227227, 1257257, 1269269, 1293293, 1347347, 1353353, 1383383, 1389389, 1419419, 1431431, 1443443, 1461461, 1467467, 1491491, 1521521 (...)

See that:

:  $11111 = 41*271$  and  $41 + 271 - 1 = 311$ , prime;  
:  $14141 = 79*179$  and  $79 + 179 - 1 = 257$ , prime;  
:  $15353 = 13*1181$  and  $13 + 1181 - 1 = 1193$ , prime;  
:  $1227227 = 163*7529$  and  $163 + 7529 - 1 = 7691$ , prime;  
:  $1383383 = 181*7643$  and  $181 + 7643 - 1 = 7823$ , prime;  
:  $1419419 = 461*3079$  and  $461 + 3079 - 1 = 3539$ , prime;  
:  $1431431 = 71*20161$  and  $71 + 20161 - 1 = 20231$ , prime.

### The sequence of squarefree composites with three prime factors:

: 17171, 1149149, 1233233, 1251251, 1359359, 1449449 (...)

See that:

:  $17171 = 7*11*223$  and  $11*223 + 7 - 1 = 2459$ , prime; also  $7*223 + 11 - 10 = 1571$ , prime;

:  $1149149 = 17 \cdot 23 \cdot 2939$  and  $17 \cdot 23 + 2939 - 1 = 3329$ , prime;  
 :  $1233233 = 19 \cdot 47 \cdot 11381$  and  $19 \cdot 47 + 1381 - 1 = 2273$ , prime;  
 :  $1251251 = 17 \cdot 89 \cdot 827$  and  $17 \cdot 89 + 827 - 1 = 2339$ , prime;  
 :  $1449449 = 29 \cdot 151 \cdot 331$  and  $29 \cdot 331 + 151 - 1 = 9749$ , prime.

**Note:**

Up to the 50-th term of the general sequence of these numbers, 21 terms are primes and 22 are semiprimes! The longest chain of consecutive terms primes met is of 5 terms: 1167167, 1173173, 1179179, 1191191, 1197197.

## 16. A method based on concatenation to create very large numbers with very few prime factors

**Abstract.** In this paper I share a very interesting discovery made more or less by accident: taking a number having just even digits, like for instance 224866802226608 (I have chosen this randomly right now when I am writing the Abstract) and concatenating it three times with itself and then to the right with the digit 1 (like in the example taken 2248668022266082248668022266082248668022266081) seems that are great chances to obtain a number with very few prime factors (in the case taken just 4 prime factors).

### Observation:

Taking a number having just even digits and concatenating it three times with itself and then to the right with the digit 1 seems that are great chances to obtain a number with very few prime factors.

### Examples:

: for 888866824 the number 8888668248888668248888668241 has 3 prime factors;  
: for 2244660800 the number 2244660800224466080022446608001 has 2 prime factors;  
: for 66624848824 the number 6662484882466624848824666248488241 has 3 prime factors;  
: for 668482848284 the number 6684828482846684828482846684828482841 has 3 prime factors;  
: for 8000024646480 the number 8000024646480800002464648080000246464801 has 3 prime factors;  
: for 22266644488044 the number 2226664448804422266644488044222666444880441 has 4 prime factors;  
: for 880008884484828 the number 8800088844848288800088844848288800088844848281 has 3 prime factors;  
: for 444666444000804 the number 4446664440008044446664440008041 has 4 prime factors;  
: for 888866640404202 the number 8888666404042028888666404042028888666404042021 has 2 prime factors; 2  
: for 888000444000404 the number 8880004440004048880004440004048880004440004041 has 3 prime factors;  
: for 666888000444606 the number 6668880004446066668880004446066668880004446061 has 4 prime factors;  
: for 222222222222222 the number 2222222222222222222222222222222222221 has 3 prime factors;  
: for 444444444444444 the number 4444444444444444444444444444444444441 has 3 prime factors;  
: for 666666666666666 the number 6666666666666666666666666666666666661 has 3 prime factors;  
: for 888888888888888 the number 8888888888888888888888888888888888881 has 4 prime factors;  
: for 24242424242424 the number 24242424242424 24242424242424 24242424242424 242424242424241 has 4 prime factors;

```

:   for 246802468024680 the number 246802468024680 246802468024680
2468024680246801 has 3 prime factors;
:   for 64646464646464 the number 646464646464646464646464646464646464646464641 is
prime.

```

The sequence of primes obtained concatenating the numbers having only even digits three times with themselves and then to the right with the digit 1 (I conjecture that this sequence has an infinity of terms):

: 2221, 4441, 6661, 2424241, 2828281, 4040401, 4242421, 6262621, 6868681, 8282821,  
2002002001, 2242242241, 2422422421, 2482482481, 2602602601, 2622622621,  
2642642641, 4044044041, 4424424421, 4824824821, 6226226221, 6266266261,  
6486486481, 6646646641, 6666666661, 6846846841, 8448448441, 8648648641,  
2004200420041, 2024202420241, 2042204220421 (...)

The longest chain, met, with consecutive terms of the general sequence of the numbers obtained like mentioned which are primes has 4 terms: 2482482481, 2602602601, 2622622621, 2642642641.

## 17. Notable observation on the squares of primes of the form $10k + 9$

**Abstract.** In this paper I conjecture that for any square of prime of the form  $p^2 = 10k + 9$ ,  $p$  greater than or equal to 7, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/(q - 1)$  is prime and, in case that this conjecture turns out not to be true, I considered three related “weaker” conjectures.

### Conjecture:

For any square of prime of the form  $p^2 = 10k + 9$ ,  $p$  greater than or equal to 7, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/(q - 1)$  is prime.

### Verifying the conjecture:

(for the first twenty primes  $p$  with the property mentioned)

- :  $p = 7$  and  $p^2 = 49$ ;  $(p^2 - 5)/4 = 11$ , prime, so  $[q, r] = [5, 11]$ ;
- :  $p = 13$  and  $p^2 = 169$ ;  $(p^2 - 5)/4 = 41$ , prime, so  $[q, r] = [5, 41]$ ;
- :  $p = 17$  and  $p^2 = 289$ ;  $(p^2 - 7)/6 = 47$ , prime, so  $[q, r] = [7, 47]$ ;
- :  $p = 23$  and  $p^2 = 529$ ;  $(p^2 - 5)/4 = 131$ , prime, so  $[q, r] = [5, 131]$ ; also  $(p^2 - 13)/12 = 43$ , prime, so  $[q, r] = [13, 43]$ ;
- :  $p = 37$  and  $p^2 = 1369$ ;  $(p^2 - 7)/6 = 227$ , prime, so  $[q, r] = [7, 227]$ ; also  $(p^2 - 13)/12 = 113$ , prime, so  $[q, r] = [13, 113]$ ;
- :  $p = 43$  and  $p^2 = 1849$ ;  $(p^2 - 5)/4 = 461$ , prime, so  $[q, r] = [5, 461]$ ; also  $(p^2 - 7)/6 = 307$ , prime, so  $[q, r] = [7, 307]$ ; also  $(p^2 - 23)/22 = 83$ , prime, so  $[q, r] = [23, 83]$ ;
- :  $p = 47$  and  $p^2 = 2209$ ;  $(p^2 - 7)/6 = 367$ , prime, so  $[q, r] = [7, 367]$ ; also  $(p^2 - 17)/16 = 137$ , prime, so  $[q, r] = [17, 137]$ ;
- :  $p = 53$  and  $p^2 = 2809$ ;  $(p^2 - 5)/4 = 701$ , prime, so  $[q, r] = [5, 701]$ ; also  $(p^2 - 7)/6 = 467$ , prime, so  $[q, r] = [7, 467]$ ; also  $(p^2 - 13)/12 = 233$ , prime, so  $[q, r] = [13, 233]$ ;
- :  $p = 67$  and  $p^2 = 4489$ ;  $(p^2 - 13)/12 = 373$ , prime, so  $[q, r] = [13, 373]$ ;
- :  $p = 73$  and  $p^2 = 5329$ ;  $(p^2 - 7)/6 = 887$ , prime, so  $[q, r] = [7, 887]$ ; also  $(p^2 - 13)/12 = 443$ , prime, so  $[q, r] = [13, 443]$ ;
- :  $p = 83$  and  $p^2 = 6889$ ;  $(p^2 - 5)/4 = 1721$ , prime, so  $[q, r] = [5, 1721]$ ; also  $(p^2 - 43)/42 = 163$ , prime, so  $[q, r] = [43, 163]$ ;
- :  $p = 97$  and  $p^2 = 9409$ ;  $(p^2 - 5)/4 = 2351$ , prime, so  $[q, r] = [5, 2351]$ ; also  $(p^2 - 7)/6 = 1567$ , prime, so  $[q, r] = [7, 1567]$ ; also  $(p^2 - 17)/16 = 587$ , prime, so  $[q, r] = [17, 587]$ ; also  $(p^2 - 43)/42 = 223$ , prime, so  $[q, r] = [43, 223]$ ;

- :  $p = 103$  and  $p^2 = 10609$ ;  $(p^2 - 13)/12 = 883$ , prime, so  $[q, r] = [13, 883]$ ;
- :  $p = 107$  and  $p^2 = 11449$ ;  $(p^2 - 5)/4 = 2861$ , prime, so  $[q, r] = [5, 2861]$ ; also  $(p^2 - 7)/6 = 1907$ , prime, so  $[q, r] = [7, 1907]$ ; also  $(p^2 - 13)/12 = 953$ , prime, so  $[q, r] = [13, 953]$ ; also  $(p^2 - 37)/36 = 317$ , prime, so  $[q, r] = [37, 317]$ ;
- :  $p = 113$  and  $p^2 = 12769$ ;  $(p^2 - 5)/4 = 3191$ , prime, so  $[q, r] = [5, 3191]$ ; also  $(p^2 - 13)/12 = 1063$ , prime, so  $[q, r] = [13, 1063]$ ; also  $(p^2 - 17)/16 = 797$ , prime, so  $[q, r] = [17, 797]$ ;
- :  $p = 127$  and  $p^2 = 16129$ ;  $(p^2 - 7)/6 = 2687$ , prime, so  $[q, r] = [7, 2687]$ ; also  $(p^2 - 43)/42 = 383$ , prime, so  $[q, r] = [43, 383]$ ; also  $(p^2 - 73)/72 = 223$ , prime, so  $[q, r] = [73, 223]$ ; also  $(p^2 - 97)/96 = 167$ , prime, so  $[q, r] = [97, 167]$ ;
- :  $p = 137$  and  $p^2 = 18769$ ;  $(p^2 - 5)/4 = 4691$ , prime, so  $[q, r] = [5, 4691]$  ;
- :  $p = 157$  and  $p^2 = 24649$ ;  $(p^2 - 13)/12 = 2053$ , prime, so  $[q, r] = [13, 2053]$ ; :  $p = 163$  and  $p^2 = 26569$ ;  $(p^2 - 13)/12 = 2213$ , prime, so  $[q, r] = [13, 2213]$ ;
- :  $p = 167$  and  $p^2 = 27889$ ;  $(p^2 - 5)/4 = 6971$ , prime, so  $[q, r] = [5, 6971]$ .

**Note:**

In case that the conjecture above turns out not to be true there are three “weaker” conjectures that may be considered:

- (i) For any square of prime of the form  $p^2 = 10k + 9$ ,  $p$  greater than or equal to 7, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/(q - 1)$  is prime or a power of prime.

Example:  $p = 73$ ,  $p^2 = 5329$ ,  $(p^2 - 5)/4 = 11^3$ .

- (ii) For any square of prime of the form  $p^2 = 10k + 9$ ,  $p$  greater than or equal to 7, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/(q - 1)$  is prime or semiprime  $m \cdot n$ ,  $n > m$ , with the property that  $n - m + 1$  is prime or power of prime or  $n + m - 1$  is prime or power of prime.

Examples:

- :  $p = 67$ ,  $p^2 = 4489$ ,  $(p^2 - 5)/4 = 19 \cdot 59$  and  $59 - 19 + 1 = 41$ ;
- :  $p = 53$ ,  $p^2 = 2809$ ,  $(p^2 - 19)/18 = 5 \cdot 31$  and  $31 - 5 + 1 = 3^3$ ;
- :  $p = 127$ ,  $p^2 = 16129$ ,  $(p^2 - 113)/112 = 11 \cdot 13$  and  $13 + 11 - 1 = 23$ .

- (iii) For any square of prime of the form  $p^2 = 10k + 9$ ,  $p$  greater than or equal to 7, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/((q - 1) \cdot 2^n)$  is prime.

Examples:

- :  $p = 113$ ,  $p^2 = 11449$ ,  $(p^2 - 73)/(72 \cdot 2) = 79$ ;
- :  $p = 137$ ,  $p^2 = 18769$ ,  $(p^2 - 17)/(16 \cdot 2^2) = 293$ ;
- :  $p = 167$ ,  $p^2 = 27889$ ,  $(p^2 - 113)/(112 \cdot 2^3) = 31$ .

## 18. Notable observation on the squares of primes of the form $10k + 1$

**Abstract.** In this paper I conjecture that for any square of prime of the form  $p^2 = 10k + 1$ ,  $p$  greater than or equal to 11, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/(q - 1)$  is prime and, in case that this conjecture turns out not to be true, I considered three related “weaker” conjectures.

### Conjecture:

For any square of prime of the form  $p^2 = 10k + 1$ ,  $p$  greater than or equal to 11, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/(q - 1)$  is prime.

### Verifying the conjecture:

(for the first ten primes  $p$  with the property mentioned)

- :  $p = 11$  and  $p^2 = 121$ ;  $(p^2 - 5)/4 = 29$ , prime;
- :  $p = 19$  and  $p^2 = 361$ ;  $(p^2 - 5)/4 = 89$ , prime; also  $(p^2 - 7)/6 = 59$ , prime; also  $(p^2 - 13)/12 = 29$ , prime;
- :  $p = 29$  and  $p^2 = 841$ ;  $(p^2 - 7)/6 = 139$ , prime; also  $(p^2 - 11)/10 = 83$ , prime;
- :  $p = 31$  and  $p^2 = 961$ ;  $(p^2 - 5)/4 = 239$ , prime; also  $(p^2 - 13)/12 = 79$ , prime; also  $(p^2 - 17)/16 = 59$ , prime;
- :  $p = 41$  and  $p^2 = 1681$ ;  $(p^2 - 5)/4 = 419$ , prime; also  $(p^2 - 11)/10 = 167$ , prime; also  $(p^2 - 13)/12 = 139$ , prime; also  $(p^2 - 29)/28 = 59$ , prime;
- :  $p = 59$  and  $p^2 = 3481$ ;  $(p^2 - 11)/10 = 347$ , prime;
- :  $p = 61$  and  $p^2 = 3721$ ;  $(p^2 - 5)/4 = 929$ , prime; also  $(p^2 - 7)/6 = 619$ , prime;
- :  $p = 71$  and  $p^2 = 5041$ ;  $(p^2 - 5)/4 = 1259$ , prime; also  $(p^2 - 7)/6 = 839$ , prime; also  $(p^2 - 11)/10 = 503$ , prime; also  $(p^2 - 13)/12 = 419$ , prime; also  $(p^2 - 29)/28 = 179$ , prime; also  $(p^2 - 31)/30 = 167$ , prime; also  $(p^2 - 37)/36 = 139$ , prime; also  $(p^2 - 61)/60 = 83$ , prime;
- :  $p = 79$  and  $p^2 = 6241$ ;  $(p^2 - 5)/4 = 1559$ , prime; also  $(p^2 - 7)/6 = 1039$ , prime; also  $(p^2 - 17)/16 = 389$ , prime; also  $(p^2 - 61)/60 = 103$ , prime;
- :  $p = 89$  and  $p^2 = 7921$ ;  $(p^2 - 5)/4 = 1979$ , prime; also  $(p^2 - 7)/6 = 1319$ , prime; also  $(p^2 - 13)/12 = 659$ , prime; also  $(p^2 - 19)/18 = 439$ , prime; also  $(p^2 - 23)/22 = 359$ , prime; also  $(p^2 - 31)/30 = 263$ , prime; also  $(p^2 - 41)/40 = 197$ , prime; also  $(p^2 - 61)/60 = 131$ , prime; also  $(p^2 - 73)/72 = 109$ , prime.

**Note:**

In case that the conjecture above turns out not to be true there are three “weaker” conjectures that may be considered:

- (i) For any square of prime of the form  $p^2 = 10k + 1$ ,  $p$  greater than or equal to 11, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/(q - 1)$  is prime or a power of prime.

Example:

$$: \quad p = 59, p^2 = 3481, (p^2 - 13)/12 = 17^2, \text{ square of prime.}$$

- (ii) For any square of prime of the form  $p^2 = 10k + 1$ ,  $p$  greater than or equal to 11, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/(q - 1)$  is prime or semiprime  $m \cdot n$ ,  $n > m$ , with the property that  $n - m + 1$  is prime or power of prime or  $n + m - 1$  is prime or power of prime.

Examples:

$$: \quad p = 61, p^2 = 3721, (p^2 - 11)/10 = 7 \cdot 53 \text{ and } 53 - 7 + 1 = 47, \text{ prime; also } 53 + 7 - 1 = 59, \text{ prime;}$$

$$: \quad p = 71, p^2 = 5041, (p^2 - 43)/42 = 7 \cdot 17 \text{ and } 17 - 7 + 1 = 11, \text{ prime; also } 17 + 7 - 1 = 23, \text{ prime.}$$

- (iii) For any square of prime of the form  $p^2 = 10k + 1$ ,  $p$  greater than or equal to 11, is true that there exist at least one prime  $q$ ,  $q$  lesser than  $p$ , such that  $r = (p^2 - q)/((q - 1) \cdot 2^n)$  is prime.

Examples:

$$: \quad p = 61, p^2 = 3721, (p^2 - 41)/(40 \cdot 2^2) = 23, \text{ prime;}$$

$$: \quad p = 71, p^2 = 5041, (p^2 - 17)/(16 \cdot 2) = 157, \text{ prime.}$$

## 19. Conjecture that states that the square of any prime can be written in a certain way

**Abstract.** In this paper we conjecture that the square of any prime greater than or equal to 5 can be written in one of the following three ways: (i)  $p*q + q - p$ ; (ii)  $p*q*r + p*q - r$ ; (iii)  $p*q*r + p - q*r$ , where  $p$ ,  $q$  and  $r$  are odd primes. Incidentally, verifying this conjecture, we found results that encouraged us to issue yet another conjecture, i.e. that the square of any prime of the form  $11 + 30*k$  can be written as  $3*p*q + p - 3*q$ , where  $p$  and  $q$  are odd primes.

### Conjecture:

The square of any prime  $s$  greater than or equal to 5 can be written in one of the following three ways: (i)  $p*q + q - p$ ; (ii)  $p*q*r + p*q - r$ ; (iii)  $p*q*r + p - q*r$ , where  $p$ ,  $q$  and  $r$  are odd primes.

### Verifying the conjecture:

(up to  $s = 41$ )

- :  $5^2 = 25 = 3*7 + 7 - 3$ ;
- :  $7^2 = 49 = 3*13 + 13 - 3$ ; also  $49 = 3*3*5 + 3*3 - 5$ ;
- :  $11^2 = 121 = 3*31 + 31 - 3$ ; also  $121 = 3*3*13 + 13 - 3*3$ ; also  $121 = 3*5*7 + 3*7 - 5$ ;
- :  $13^2 = 169 = 5*29 + 29 - 5$ ; also  $169 = 3*43 + 43 - 3$ ; also  $169 = 3*5*11 + 3*5 - 11$ ;
- :  $17^2 = 289 = 7*37 + 37 - 7$ ; also  $289 = 3*5*19 + 19 - 3*5$ ; also  $289 = 5*5*11 + 5*5 - 11$ ; also  $289 = 5*7*7 + 7*7 - 5$ ;
- :  $19^2 = 361 = 11*31 + 31 - 11$ ; also  $361 = 3*7*17 + 3*7 - 17$ ; also  $361 = 3*3*37 + 37 - 3*3$ ;
- :  $23^2 = 529 = 7*67 + 67 - 7$ ; also  $529 = 5*89 + 89 - 5$ ;
- :  $29^2 = 841 = 19*43 + 43 - 19$ ; also  $841 = 13*61 + 61 - 13$ ; also  $841 = 11*71 + 71 - 11$ ;
- :  $31^2 = 961 = 23*41 + 41 - 23$ ; also  $961 = 3*11*29 + 3*11 - 29$ ; also  $961 = 7*7*19 + 7*7 - 19$ ; also  $961 = 3*5*61 + 61 - 3*5$ ;
- :  $37^2 = 1369 = 7*11*17 + 7*11 - 17$ ;
- :  $41^2 = 1681 = 23*71 + 71 - 23$ ; also  $1681 = 3*13*43 + 43 - 3*13$ ; also  $1681 = 3*19*29 + 3*19 - 29$ ; also  $1681 = 5*17*19 + 5*17 - 19$ .

**Conjecture:**

The square of any prime  $s$  of the form  $11 + 30 \cdot k$  can be written as  $3 \cdot p \cdot q + p - 3 \cdot q$ , where  $p$  and  $q$  are odd primes.

**Verifying the conjecture:**

(up to  $s = 131$ )

- : for  $s = 11$  we have  $[p, q] = [13, 3]$  (see above);
- : for  $s = 41$  we have  $[p, q] = [43, 13]$  (see above);
- : for  $s = 71$  we have  $[p, q] = [73, 23]$ ;
- : for  $s = 101$  we have  $[p, q] = [1021, 3]$  and  $[31, 113]$ ;
- : for  $s = 131$  we have  $[p, q] = [331, 17]$ ,  $[79, 73]$  and  $[953, 7]$ ;
- : for  $s = 191$  we have  $[p, q] = [2281, 5]$ ,  $[229, 53]$  and  $[13, 1013]$ .

## 20. Conjecture on the numbers $(p^2 - n)/(n - 1)$ where $p$ prime

**Abstract.** In this paper I state the following conjecture: for any  $p$  prime there exist at least a value of  $n$ , different from  $p$ , for which the number  $(p^2 - n)/(n - 1)$  is prime.

### Conjecture:

For any  $p$  prime there exist at least a value of  $n$ , different from  $p$ , for which the number  $q = (p^2 - n)/(n - 1)$  is prime.

### Verifying the conjecture:

(for the first 7 primes  $p$ )

- : for  $p = 5$ ,  $q = 23$ , prime, for  $n = 2$ ; also  $q = 11$ , prime for  $n = 3$ ; also  $q = 7$ , prime, for  $n = 4$ ;
- : for  $p = 7$ ,  $q = 47$ , prime, for  $n = 2$ ; also  $q = 23$ , prime, for  $n = 3$ ; also  $q = 11$ , prime, for  $n = 5$ ;
- : for  $p = 11$ ,  $q = 29$ , prime, for  $n = 5$ ; also  $q = 23$ , prime for  $n = 6$ ; also  $q = 19$ , prime, for  $n = 7$ ;
- : for  $p = 13$ ,  $q = 167$ , prime, for  $n = 2$ ; also  $q = 83$ , prime, for  $n = 3$ ; also  $q = 4$ , prime, for  $n = 5$ ; also  $q = 23$ , prime, for  $n = 8$ ;
- : for  $p = 17$ ,  $q = 71$ , prime, for  $n = 5$ ; also  $q = 47$ , prime, for  $n = 7$ ; also  $q = 31$ , prime, for  $n = 10$ ; also  $q = 23$ , prime, for  $n = 13$ ;
- : for  $p = 19$ ,  $q = 359$ , prime, for  $n = 1$ ; also  $q = 179$ , prime, for  $n = 3$ ; also  $q = 89$ , prime, for  $n = 5$ ; also  $q = 71$ , prime, for  $n = 6$ ; also  $q = 59$ , prime, for  $n = 7$ ; also  $q = 29$ , prime, for  $n = 13$ ; also  $q = 23$ , prime, for  $n = 16$ ;
- : for  $p = 23$ ,  $q = 263$ , prime, for  $n = 3$ ; also  $q = 131$ , prime, for  $n = 5$ ; also  $q = 47$ , prime, for  $n = 12$ ; also  $q = 43$ , prime, for  $n = 13$ .

Note that many primes (I conjecture that an infinity of primes) can be written as  $\text{sqr}(24*m - 23)$ :

- :  $7 = \text{sqr}(24*3 - 23)$ ;
- :  $11 = \text{sqr}(24*6 - 23)$ ;
- :  $13 = \text{sqr}(24*8 - 23)$ ;
- :  $17 = \text{sqr}(24*13 - 23)$ ;
- :  $19 = \text{sqr}(24*16 - 23)$ .

I also conjecture that there exist an infinity of primes that can be written as  $\text{sqr}(48*m - 47)$ ; examples: 7, 17, 23 for  $n = 2, 7, 12$ .

## 21. Conjecture on the numbers $3p(q - 1) - 1$ where $p$ and $q$ are primes and $p = q + 6$

**Abstract.** In this paper I state the following conjecture: there exist an infinity of primes of the form  $3p(q - 1) - 1$ , where  $p$  and  $q$  are primes and  $p = q + 6$ . Note that from the first terms of the sequence of sexy primes we have a chain of consecutive 9 primes: 131, 233, 509, 683, 1103, 1913, 3329, 4643, 5639 (for  $q = 5, 7, 11, 13, 17, 23, 31, 37, 41$ ).

### Conjecture:

There exist an infinity of primes of the form  $3p(q - 1) - 1$ , where  $p$  and  $q$  are primes and  $p = q + 6$ . Note that from the first terms of the sequence of sexy primes we have a chain of consecutive 9 primes: 131, 233, 509, 683, 1103, 1913, 3329, 4643, 5639 (for  $q = 5, 7, 11, 13, 17, 23, 31, 37, 41$ ).

### The sequence of primes of this form:

:  $3 \cdot 11 \cdot (5 - 1) = 131$ , prime;  
:  $3 \cdot 13 \cdot (7 - 1) = 233$ , prime;  
:  $3 \cdot 17 \cdot (11 - 1) = 509$ , prime;  
:  $3 \cdot 19 \cdot (13 - 1) = 683$ , prime;  
:  $3 \cdot 23 \cdot (17 - 1) = 1103$ , prime;  
:  $3 \cdot 29 \cdot (23 - 1) = 1913$ , prime;  
:  $3 \cdot 37 \cdot (31 - 1) = 3329$ , prime;  
:  $3 \cdot 43 \cdot (37 - 1) = 4643$ , prime;  
:  $3 \cdot 47 \cdot (41 - 1) = 5639$ , prime;  
:  $3 \cdot 59 \cdot (53 - 1) = 9203$ , prime;  
:  $3 \cdot 89 \cdot (83 - 1) = 21893$ , prime;  
:  $3 \cdot 103 \cdot (97 - 1) = 29663$ , prime;  
:  $3 \cdot 107 \cdot (101 - 1) = 32099$ , prime;  
:  $3 \cdot 109 \cdot (103 - 1) = 33353$ , prime;  
:  $3 \cdot 113 \cdot (107 - 1) = 35933$ , prime;  
:  $3 \cdot 163 \cdot (157 - 1) = 76283$ , prime;  
:  $3 \cdot 179 \cdot (173 - 1) = 92363$ , prime;  
:  $3 \cdot 197 \cdot (191 - 1) = 112289$ , prime;  
:  $3 \cdot 257 \cdot (251 - 1) = 192749$ , prime;  
:  $3 \cdot 269 \cdot (263 - 1) = 211433$ , prime;  
:  $3 \cdot 283 \cdot (277 - 1) = 224369$ , prime;  
:  $3 \cdot 313 \cdot (307 - 1) = 287333$ , prime;  
:  $3 \cdot 317 \cdot (311 - 1) = 294809$ , prime;  
:  $3 \cdot 359 \cdot (353 - 1) = 379103$ , prime;  
:  $3 \cdot 449 \cdot (443 - 1) = 595373$ , prime;  
:  $3 \cdot 463 \cdot (457 - 1) = 595373$ , prime;  
:  $3 \cdot 509 \cdot (503 - 1) = 766553$ , prime;  
(...)

**Note:**

The sequence of the semiprimes  $m \cdot n$  of this form is also interesting because of a property shared by many of these, i.e. that  $m + n - 1$  is prime; examples:

- :  $3 \cdot 53 \cdot (47 - 1) = 7313 = 71 \cdot 103$  and  $71 + 103 - 1 = 173$ , prime;
- :  $3 \cdot 67 \cdot (61 - 1) = 12059 = 31 \cdot 389$  and  $31 + 389 - 1 = 419$ , prime;
- :  $3 \cdot 79 \cdot (73 - 1) = 17063 = 113 \cdot 151$  and  $113 + 151 - 1 = 263$ , prime;
- :  $3 \cdot 173 \cdot (167 - 1) = 86153 = 101 \cdot 853$  and  $101 + 853 - 1 = 953$ , prime;
- :  $3 \cdot 277 \cdot (271 - 1) = 224369 = 89 \cdot 2521$  and  $89 + 2521 - 1 = 2609$ , prime.

## 22. Four conjectures on the numbers obtained concatenating to the right a prime with the digit 9

**Abstract.** In this paper I state the following four conjectures: (I) There exist an infinity of primes  $p$  which, concatenated to the right with the digit 9, form also prime numbers; (II) There exist an infinity of primes obtained concatenating the reversal of  $p$  as is defined in Conjecture I to the right with the digit 9; (III) There exist an infinity of semiprimes obtained concatenating primes to the right with the digit 9, semiprimes  $m*n$  having the property that  $n - m + 1$  is prime; (IV) There exist an infinity of semiprimes obtained concatenating the reversal of  $p$  as is defined in Conjecture I to the right with the digit 9, semiprimes  $m*n$  having the property that  $n - m + 1$  is prime.

### Conjecture I:

There exist an infinity of primes  $p$  which, concatenated to the right with the digit 9, form also prime numbers  $q$ .

The sequence of primes  $q$ :

: 59, 79, 139, 179, 199, 239, 379, 419, 439, 479, 599, 619, 719, 739, 839, 1019, 1039, 1279, 1319, 1399, 1499, 1579, 1979, 1999, 2239, 2339, 2399, 2579, 2699, 2719, 2819, 2939, 3079, 3119, 3319, 3499, 3539, 3739, 4019, 4099, 4219 (...)

### Conjecture II:

There exist an infinity of primes  $r$  obtained concatenating the reversal of  $p$  as is defined in Conjecture I to the right with the digit 9.

The sequence of primes  $q$ :

: 719, 919, 739, 149, 349, 179, 379, 389, 1019, 3019, 7219, 1319, 9319, 9419, 7919, 3229, 3329, 7529, 9629, 3929, 7039, 9439, 3539, 3739, 1049, 9049, 1249 (...)

### Conjecture III:

There exist an infinity of semiprimes obtained concatenating primes to the right with the digit 9, semiprimes  $m*n$  having the property that  $n - m + 1$  is prime.

The sequence of semiprimes  $m*n$ :

: 119 (=  $7*17$  and  $17 - 7 + 1 = 11$ , prime), 299 (=  $13*23$  and  $23 - 13 + 1 = 11$ , prime), 319 (=  $11*29$  and  $29 - 11 + 1 = 19$ , prime), 799 (=  $17*47$  and  $47 - 17 + 1 = 31$ , prime), 899 (=  $29*31$  and  $31 - 29 + 1 = 3$ , prime), 979 (=  $11*89$  and  $89 - 11 + 1 = 79$ , prime), 1079 (=  $13*83$  and  $83 - 13 + 1 = 73$ , prime), 1099 (=  $7*157$  and  $157 - 7 + 1 = 151$ , prime), 1379 (=  $7*197$  and  $197 - 7 + 1 = 191$ , prime), 1639 (=  $11*149$  and  $149 - 11 + 1 = 139$ , prime), 1739 (=  $37*47$  and  $47 - 37 + 1 = 11$ , prime), 1799 (=  $7*257$  and  $257 - 7 + 1 = 251$ , prime), 1919 (=  $19*101$  and  $101 - 19 + 1 = 83$ , prime), 1939 (=  $7*277$  and  $277 - 7 + 1 = 271$ , prime), 2119 (=  $13*163$  and  $163 - 13 + 1 = 151$ , prime), 2279 (=  $43*53$  and

53 – 43 + 1 = 11, prime), 2419 (= 41\*59 and 59 – 41 + 1 = 19, prime), 2839 (= 17\*167 and 167 – 17 + 1 = 151, prime), 3139 (= 43\*73 and 73 – 43 + 1 = 31, prime), 3379 (= 31\*109 and 109 – 31 + 1 = 79, prime), 3599 (= 59\*61 and 61 – 59 + 1 = 3, prime), 3679 (= 13\*283 and 283 – 13 + 1 = 271, prime), 3799 (= 29\*131 and 131 – 29 + 1 = 103, prime), 3979 (= 23\*173 and 173 – 23 + 1 = 151, prime)...

#### **Conjecture IV:**

There exist an infinity of semiprimes  $m*n$  obtained concatenating the reversal of  $p$  as is defined in Conjecture I to the right with the digit 9, semiprimes having the property that  $n - m + 1$  is prime.

The sequence of semiprimes  $m*n$ :

: 319 (= 11\*29 and 29 – 11 + 1 = 19, prime), 329 (= 7\*47 and 47 – 7 + 1 = 41, prime), 749 (= 7\*107 and 107 – 7 + 1 = 101, prime), 959 (= 7\*137 and 137 – 7 + 1 = 131, prime), 7519 (= 73\*103 and 103 – 73 + 1 = 31, prime), 1829 (= 31\*59 and 59 – 31 + 1 = 29, prime)...

## 23. Three conjectures on the numbers obtained concatenating to the left the odd numbers with 1234

**Abstract.** In this paper I state the following three conjectures on the numbers obtained concatenating to the left the odd numbers with 1234: (I) There exist an infinity of primes obtained concatenating to the left odd numbers with 1234; (II) There exist an infinity of primes obtained concatenating to the left prime numbers with 1234; (III) There exist an infinity of primes obtained concatenating to the left Poulet numbers with 1234.

### Conjecture 1:

There exist an infinity of primes obtained concatenating to the left odd numbers with 1234.

**The sequence of these primes:**

: 12343, 12347, 123419, 123427, 123433, 123439, 123449, 123457, 123479, 123491, 123493, 123499, 1234109, 1234117, 1234133, 1234147, 1234187, 1234231, 1234237, 1234241, 1234243, 1234253, 1234271, 1234309, 1234333, 1234349, 1234351, 1234367, 1234379, 1234391, 1234393, 1234439, 1234463, 1234511, 1234517, 1234531, 1234537, 1234543, 1234547, 1234577, 1234603, 1234613 (...)

### Conjecture 2:

There exist an infinity of primes obtained concatenating to the left prime numbers with 1234.

**The sequence of these primes:**

: 12343, 12347, 123419, 123479, 123409, 1234133, 1234241, 1234271, 1234349, 1234379, 1234439, 1234463, 1234547, 1234577, 1234613 (...)

### Conjecture 3:

There exist an infinity of primes obtained concatenating to the left Poulet numbers with 1234.

**The sequence of these primes:**

: 12341729, 12342047, 12342821, 12344681, 12346601, 123412801, 123413747, 123415709, 123415841, 123418721, 123419951, 123433153 (...)

## 24. Conjecture on the primes obtained deconcatenating to the right the numbers $(30k-1)(30k+1)$ with digit 9

**Abstract.** In this paper I state the following conjecture: there exist an infinity of primes obtained deconcatenating the numbers of the form  $(30*k - 1)*(30*k + 1)$  to the right with digit 9; example:  $449*451 = 202499$  and 20249 is a prime.

### Conjecture :

There exist an infinity of primes  $p$  obtained deconcatenating the numbers of the form  $(30*k - 1)*(30*k + 1)$  to the right with digit 9.

The sequence of primes  $p$ :

:  $29*31 = 899$  and  $p = 89$  is prime;  
:  $59*61 = 3599$  and  $p = 359$  is prime;  
:  $89*91 = 8099$  and  $p = 809$  is prime;  
:  $119*121 = 14399$  and  $p = 1439$  is prime;  
:  $209*211 = 44099$  and  $p = 4409$  is prime;  
:  $299*301 = 89999$  and  $p = 8999$  is prime;  
:  $329*331 = 108899$  and  $p = 10889$  is prime;  
:  $359*361 = 129599$  and  $p = 12959$  is prime;  
:  $449*451 = 202499$  and  $p = 20249$  is prime;  
:  $479*481 = 230399$  and  $p = 23039$  is prime;  
:  $599*601 = 359999$  and  $p = 35999$  is prime;  
:  $689*691 = 476099$  and  $p = 47609$  is prime;  
:  $719*721 = 518399$  and  $p = 51839$  is prime;  
:  $749*751 = 562499$  and  $p = 56249$  is prime;  
:  $809*811 = 656099$  and  $p = 65609$  is prime;  
:  $869*871 = 756899$  and  $p = 75689$  is prime;  
:  $989*991 = 980099$  and  $p = 98009$  is prime;  
:  $1019*1021 = 1040399$  and  $p = 104039$  is prime;  
:  $1079*1081 = 1166399$  and  $p = 116639$  is prime;  
:  $2009*2011 = 4040099$  and  $p = 404009$  is prime;  
:  $2039*2041 = 4161599$  and  $p = 416159$  is prime;  
:  $2069*2071 = 4284899$  and  $p = 428489$  is prime;  
:  $2219*2221 = 4928399$  and  $p = 492839$  is prime;  
:  $2339*2341 = 5475599$  and  $p = 547559$  is prime;  
:  $2429*2431 = 5904899$  and  $p = 590489$  is prime;  
:  $2519*2521 = 6350399$  and  $p = 635039$  is prime;  
:  $2669*2671 = 7128899$  and  $p = 712889$  is prime;  
:  $2819*2821 = 7952399$  and  $p = 795239$  is prime;  
:  $2849*2851 = 8122499$  and  $p = 812249$  is prime;  
:  $2939*2941 = 8643599$  and  $p = 864359$  is prime;  
:  $3029*3031 = 9180899$  and  $p = 918089$  is prime;  
:  $3119*3121 = 9734399$  and  $p = 973439$  is prime.

(...)

## 25. Two formulae for obtaining primes based on the prime decomposition of the number 561

**Abstract.** In this paper I present two formulae which seems to conduct to primes or products of very few prime factors, both of them inspired by the prime decomposition of the first absolute Fermat pseudoprime, the number 561.

### Formula I

#### Observation:

Noting that the number  $N = 561 = 3 \cdot 11 \cdot 17$  has the property that conducts to a prime for two values of  $d$  from three, where  $d$  prime factor, through the formula  $N - N/d - 1$  (i.e.  $373 = 561 - 561/3 - 1$  and  $509 = 561 - 561/11 - 1$ ), I wondered if it is a general property of the numbers of the form  $N = 3 \cdot p \cdot q$ , where  $(p, q)$  is a pair of sexy primes, to conduct often to primes and products of very few prime factors and it seems that, indeed, it is.

#### Verifying the observation:

(For the first 34 pairs of sexy primes)

- : for  $(p, q) = (5, 11)$  are obtained the primes 109, 131 and 149;
- : for  $(p, q) = (7, 13)$  are obtained the primes 181, 233 and 251;
- : for  $(p, q) = (11, 17)$  are obtained the primes 373 and 509;
- : for  $(p, q) = (13, 19)$  are obtained the primes 683 and 701;
- : for  $(p, q) = (17, 23)$  is obtained the prime 1103;
- : for  $(p, q) = (23, 29)$  are obtained the primes 1913 and 1931;
- : for  $(p, q) = (31, 37)$  are obtained the primes 2293 and 3329;
- : for  $(p, q) = (37, 43)$  are obtained the primes 3181 and 4643;
- : for  $(p, q) = (47, 53)$  is obtained the prime 7331;
- : for  $(p, q) = (53, 59)$  are obtained the primes 9203 and 9221;
- : for  $(p, q) = (53, 59)$  are obtained the primes 9203 and 9221;
- : for  $(p, q) = (67, 73)$  is obtained the prime 9781;
- : for  $(p, q) = (83, 89)$  are obtained the primes 21893 and 21911;
- : for  $(p, q) = (97, 103)$  is obtained the prime 29663;
- : for  $(p, q) = (101, 107)$  are obtained the primes 21613, 32099 and 32117;
- : for  $(p, q) = (103, 109)$  are obtained the primes 22453 and 33353;
- : for  $(p, q) = (107, 113)$  are obtained the primes 24181, 35933 and 35951;
- : for  $(p, q) = (151, 157)$  is obtained the prime 70667;
- : for  $(p, q) = (157, 163)$  is obtained the prime 76283;
- : for  $(p, q) = (167, 173)$  are obtained the primes 57781 and 86171;
- : for  $(p, q) = (173, 179)$  are obtained the primes 61933, 92363 and 92381;
- : for  $(p, q) = (191, 197)$  are obtained the primes 75253 and 111697;
- : for  $(p, q) = (193, 199)$  is obtained the prime 114641;
- : for  $(p, q) = (223, 229)$  is obtained the prime 152531;
- : for  $(p, q) = (227, 233)$  is obtained the prime 157991;
- : for  $(p, q) = (233, 239)$  is obtained the prime 111373;
- : for  $(p, q) = (251, 257)$  are obtained the primes 192749 and 192767;
- : for  $(p, q) = (257, 263)$  are obtained the primes 135181 and 202001;
- : for  $(p, q) = (263, 269)$  is obtained the prime 211433;



## 26. Four conjectures on the numbers created concatenating the product of twin primes with 11

**Abstract.** In this paper I make four conjectures on the numbers  $n$  created concatenating to the right the product  $p*q$  with number 11, where  $[p, q]$  is a pair of twin primes: (I) there exist an infinity of  $n$  primes; (II) there exist an infinity of  $n$  semiprimes of the form  $(10k + 1)*(10h + 1)$ ; (II) there exist an infinity of  $n$  semiprimes of the form  $(10k + 9)*(10h + 9)$ ; (II) there exist an infinity of  $n$  semiprimes of the form  $(10k + 3)*(10h + 7)$ . Note that for 40 from the first 43 pairs of twin primes the number  $n$  belongs to one of the four sequences considered by the conjectures above.

### Conjecture I:

There exist an infinity of primes created concatenating to the right the product  $p*q$  with number 11, where  $[p, q]$  is a pair of twin primes.

Example: for the pair of twin primes  $[p, q] = [59, 61]$  the product  $p*q = 3599$ ; concatenating this number to the right with 11 is obtained the number 359911, prime.

#### The sequence of these primes:

: 1511, 3511, 359911, 518311, 1040311, 1166311, 1904311, 2249911, 3920311, 5759911, 7289911, 12110311, 17639911, 21344311, 27248311, 32489911, 38192311, 43559911, 65768311, 68558311, 77792311, 132710311  
(...)  
obtained for  $[p, q] = [3, 5], [5, 7], [59, 61], [71, 73], [101, 103], [107, 109], [137, 139], [149, 151], [197, 199], [239, 241], [269, 271], [347, 349], [419, 421], [461, 463], [521, 523], [569, 571], [617, 619], [659, 661], [821, 823], [827, 829], [881, 883], [1151, 1153]$ .

Note the chain of six primes obtained for six consecutive pairs of twin primes: 359911, 518311, 1040311, 1166311, 1904311, 2249911.

### Conjecture II:

There exist an infinity of semiprimes  $n$  of the form  $(10k + 1)*(10h + 1)$  created concatenating to the right the product  $p*q$  with number 11, where  $[p, q]$  is a pair of twin primes.

#### The sequence of these semiprimes:

:  $n = 14311 = 11*1301$  for  $[p, q] = [11, 13]$ ;  
:  $n = 65768311 = 1291*50821$  for  $[p, q] = [809, 811]$ ;  
:  $n = 104039911 = 631*164881$  for  $[p, q] = [1019, 1021]$ ;  
:  $n = 119246311 = 5741*20771$  for  $[p, q] = [1091, 1093]$ .

### Conjecture III:

There exist an infinity of semiprimes  $n$  of the form  $(10k + 9)(10h + 9)$  created concatenating to the right the product  $p*q$  with number 11, where  $[p, q]$  is a pair of twin primes.

#### The sequence of these semiprimes:

- :  $n = 32311 = 79*409$  for  $[p, q] = [17, 19]$ ;
- :  $n = 106502311 = 3989*26699$  for  $[p, q] = [1031, 1033]$ ;
- :  $n = 151289911 = 1019*148469$  for  $[p, q] = [1229, 1231]$ ;
- :  $n = 1634432311 = 229*7137259$  for  $[p, q] = [1277, 1279]$ .

### Conjecture IV:

There exist an infinity of semiprimes  $n$  of the form  $(10k + 3)(10h + 7)$  created concatenating to the right the product  $p*q$  with number 11, where  $[p, q]$  is a pair of twin primes.

#### The sequence of these semiprimes:

- :  $n = 89911 = 47*1913$  for  $[p, q] = [29, 31]$ ;
- :  $n = 176311 = 157*1123$  for  $[p, q] = [41, 43]$ ;
- :  $n = 3239911 = 17*190583$  for  $[p, q] = [179, 181]$ ;
- :  $n = 3686311 = 607*6073$  for  $[p, q] = [191, 193]$ ;
- :  $n = 5198311 = 17*305783$  for  $[p, q] = [227, 229]$ ;
- :  $n = 7952311 = 17*467783$  for  $[p, q] = [281, 283]$ ;
- :  $n = 9734311 = 47*207113$  for  $[p, q] = [311, 313]$ ;
- :  $n = 18662311 = 17*1097783$  for  $[p, q] = [431, 433]$ ;
- :  $n = 41216311 = 73*564607$  for  $[p, q] = [641, 643]$ ;
- :  $n = 112784311 = 2803*40237$  for  $[p, q] = [1061, 1063]$ .

### Note:

For 40 from the first 43 pairs of twin primes the number  $n$  belongs to one of the four sequences considered by the conjectures above.

## 27. Two conjectures on the numbers created concatenating an odd $n$ with $3n-4$ and then with 1 or 11

**Abstract.** In this paper I make two conjectures on the numbers  $m$  created concatenating to the right an odd number  $n$ , not divisible by 3, with  $3*n - 4$  and then, if  $n$  is of the form  $6*k + 1$ , with 11, respectively, if  $n$  is of the form  $6*k - 1$ , with 1: (I) there exist an infinity of  $m$  primes; (II) there exist an infinity of  $m = p*q$  composites such that  $p + q - 1$  is prime (where  $p$  and  $q$  may be, or may be not, primes). Note that for 25 from the first 30 odd numbers  $n$  not divisible by 3 the number  $m$  obtained belongs to one of the two sequences considered by the conjectures above.

### Conjecture I:

There exist an infinity of primes  $m$  created concatenating to the right an odd number  $n$ , not divisible by 3, with  $3*n - 4$  and then, if  $n$  is of the form  $6*k + 1$ , with 11, respectively, if  $n$  is of the form  $6*k - 1$ , with 1.

Examples:

- : for  $n = 7$  (of the form  $6*k + 1$ ), we have  $3*n - 4 = 17$  and  $m = 71711$ , prime;
- : for  $n = 17$  (of the form  $6*k - 1$ ), we have  $3*n - 4 = 47$  and  $m = 17471$ , prime.

#### The sequence of the primes $m$ :

- : 71711, 17471, 318911, 351011, 531551, 832451, 952811
- (...)
- obtained for  $n = 7, 17, 31, 35, 53, 83, 95$ .

### Conjecture II:

There exist an infinity of composites  $m = p*q$ , with the property that  $p + q - 1$  is prime (where  $p$  and  $q$  may be, or may be not, primes), created concatenating to the right an odd number  $n$ , not divisible by 3, with  $3*n - 4$  and then, if  $n$  is of the form  $6*k + 1$ , with 11, respectively, if  $n$  is of the form  $6*k - 1$ , with 1.

#### The sequence of the composites $m$ :

- :  $m = 11291$  (for  $n = 11$ ) =  $7*1613$  and  $7 + 1613 - 1 = 1619$ , prime;
- :  $m = 133511$  (for  $n = 13$ ) =  $7*19073$  and  $7 + 19073 - 1 = 19079$ , prime;
- :  $m = 23651$  (for  $n = 23$ ) =  $67*353$  and  $67 + 353 - 1 = 419$ , prime;
- :  $m = 257111$  (for  $n = 25$ ) =  $41*6271$  and  $41 + 6271 - 1 = 6311$ , prime;
- :  $m = 29831$  (for  $n = 29$ ) =  $23*1297$  and  $23 + 1297 - 1 = 1319$ , prime;
- :  $m = 411191$  (for  $n = 41$ ) =  $29*14179$  and  $29 + 14179 - 1 = 14207$ , prime; also  $411191 = 319*1289$  and  $319 + 1289 - 1 = 1607$ , prime;
- :  $m = 4312511$  (for  $n = 43$ ) =  $7*616073$  and  $7 + 616073 - 1 = 616079$ , prime;
- :  $m = 5516111$  (for  $n = 55$ ) =  $1231*4481$  and  $1231 + 4481 - 1 = 5711$ , prime;
- :  $m = 6117911$  (for  $n = 61$ ) =  $43*142277$  and  $43 + 142277 - 1 = 142319$ , prime; also  $6117911 = 1949*3139$  and  $1949 + 3139 - 1 = 5087$ , prime;

:  $m = 65191$  (for  $n = 65$ ) =  $7 \cdot 9313$  and  $7 + 9313 - 1 = 9319$ , prime; also  $6117911 = 67 \cdot 973$  and  $67 + 973 - 1 = 1039$ , prime; also  $6117911 = 139 \cdot 469$  and  $139 + 469 - 1 = 607$ , prime;  
 :  $m = 6719711$  (for  $n = 67$ ) =  $19 \cdot 353687$  and  $19 + 353687 - 1 = 353687$ , prime; also  $6719711 = 53 \cdot 126787$  and  $53 + 126787 - 1 = 126839$ , prime;  
 :  $m = 712091$  (for  $n = 71$ ) =  $509 \cdot 1399$  and  $509 + 1399 - 1 = 1907$ , prime;  
 :  $m = 772271$  (for  $n = 77$ ) =  $23 \cdot 33577$  and  $23 + 33577 - 1 = 33599$ , prime;  
 :  $m = 7923311$  (for  $n = 79$ ) =  $11 \cdot 720301$  and  $11 + 720301 - 1 = 720311$ , prime;  
 :  $m = 8525111$  (for  $n = 85$ ) =  $23 \cdot 370657$  and  $23 + 370657 - 1 = 370679$ , prime;  
 :  $m = 9728711$  (for  $n = 97$ ) =  $2749 \cdot 3539$  and  $2749 + 3539 - 1 = 6287$ , prime;  
 :  $m = 1012991$  (for  $n = 101$ ) =  $7 \cdot 144713$  and  $7 + 144713 - 1 = 144719$ , prime; also  $1012991 = 47 \cdot 21553$  and  $47 + 21553 - 1 = 21599$ , prime; also  $1012991 = 329 \cdot 3079$  and  $329 + 3079 - 1 = 21599$ , prime.

**Note:**

For 25 from the first 30 odd numbers  $n$  not divisible by 3 the number  $m$  obtained belongs to one of the two sequences considered by the conjectures above.

## 28. Two unusual defined and possible infinite sequences of primes and two conjectures

**Abstract.** In this paper I state two conjectures: (I) There exist an infinity of primes  $p$  of the form  $n - 1$ , where  $n$  is the number obtained concatenating the digits of a prime  $q$ , each one of them multiplied by 6 (example: for  $q = 239$ ,  $n = 121854$  and  $p = n - 1 = 121853$ , prime); (II) There exist an infinity of primes  $p$  of the form  $n + 1$ , where  $n$  is the number obtained concatenating the digits of a prime  $q$ , each one of them multiplied by 6 (example: for  $q = 283$ ,  $n = 124818$  and  $p = n + 1 = 124819$ , prime).

### Conjecture 1:

There exist an infinity of primes  $p$  of the form  $n - 1$ , where  $n$  is the number obtained concatenating the digits of a prime  $q$ , each one of them multiplied by 6.

Example: for  $q = 239$ ,  $n = 121854$  and  $p = 121853$ , prime.

The sequence of primes  $p$ :

: for  $p = 5$ ,  $n = 30$  and  $p = n - 1 = 29$ , prime;  
: for  $p = 7$ ,  $n = 42$  and  $p = n - 1 = 41$ , prime;  
: for  $p = 13$ ,  $n = 618$  and  $p = n - 1 = 617$ , prime;  
: for  $p = 17$ ,  $n = 642$  and  $p = n - 1 = 641$ , prime;  
: for  $p = 23$ ,  $n = 1218$  and  $p = n - 1 = 1217$ , prime;  
: for  $p = 43$ ,  $n = 2418$  and  $p = n - 1 = 2417$ , prime;  
: for  $p = 47$ ,  $n = 2442$  and  $p = n - 1 = 2441$ , prime;  
: for  $p = 73$ ,  $n = 4218$  and  $p = n - 1 = 4217$ , prime;  
: for  $p = 79$ ,  $n = 4254$  and  $p = n - 1 = 4253$ , prime;  
: for  $p = 83$ ,  $n = 4818$  and  $p = n - 1 = 4817$ , prime;  
: for  $p = 97$ ,  $n = 5442$  and  $p = n - 1 = 5441$ , prime;  
: for  $p = 109$ ,  $n = 6054$  and  $p = n - 1 = 6053$ , prime;  
: for  $p = 163$ ,  $n = 63618$  and  $p = n - 1 = 63617$ , prime;  
: for  $p = 173$ ,  $n = 64218$  and  $p = n - 1 = 64217$ , prime;  
: for  $p = 239$ ,  $n = 121854$  and  $p = n - 1 = 121853$ , prime;  
: for  $p = 269$ ,  $n = 123654$  and  $p = n - 1 = 123653$ , prime;  
: for  $p = 307$ ,  $n = 18042$  and  $p = n - 1 = 18041$ , prime;  
: for  $p = 313$ ,  $n = 18618$  and  $p = n - 1 = 18617$ , prime;  
: for  $p = 349$ ,  $n = 182454$  and  $p = n - 1 = 182453$ , prime;  
: for  $p = 397$ ,  $n = 185442$  and  $p = n - 1 = 185441$ , prime;  
(...)  
: for  $p = 104717$ ,  $n = 602442642$  and  $p = n - 1 = 602442641$ , prime;  
(...)

### Conjecture 2:

There exist an infinity of primes  $p$  of the form  $n + 1$ , where  $n$  is the number obtained concatenating the digits of a prime  $q$ , each one of them multiplied by 6.

Example: for  $q = 283$ ,  $n = 124818$  and  $p = 124819$ , prime.

The sequence of primes  $p$ :

```
:   for  $p = 5$ ,  $n = 30$  and  $p = n + 1 = 31$ , prime;
:   for  $p = 7$ ,  $n = 42$  and  $p = n + 1 = 43$ , prime;
:   for  $p = 11$ ,  $n = 66$  and  $p = n + 1 = 67$ , prime;
:   for  $p = 13$ ,  $n = 618$  and  $p = n + 1 = 619$ , prime;
:   for  $p = 17$ ,  $n = 642$  and  $p = n + 1 = 643$ , prime;
:   for  $p = 53$ ,  $n = 3018$  and  $p = n + 1 = 3019$ , prime;
:   for  $p = 61$ ,  $n = 366$  and  $p = n + 1 = 367$ , prime;
:   for  $p = 67$ ,  $n = 3642$  and  $p = n + 1 = 3643$ , prime;
:   for  $p = 73$ ,  $n = 4218$  and  $p = n + 1 = 4219$ , prime;
:   for  $p = 97$ ,  $n = 5442$  and  $p = n + 1 = 5443$ , prime;
:   for  $p = 101$ ,  $n = 606$  and  $p = n + 1 = 607$ , prime;
:   for  $p = 107$ ,  $n = 6042$  and  $p = n + 1 = 6043$ , prime;
:   for  $p = 113$ ,  $n = 6618$  and  $p = n + 1 = 6619$ , prime;
:   for  $p = 137$ ,  $n = 61842$  and  $p = n + 1 = 61843$ , prime;
:   for  $p = 191$ ,  $n = 6546$  and  $p = n + 1 = 6547$ , prime;
:   for  $p = 193$ ,  $n = 65418$  and  $p = n + 1 = 65419$ , prime;
:   for  $p = 263$ ,  $n = 123618$  and  $p = n + 1 = 123619$ , prime;
:   for  $p = 281$ ,  $n = 12486$  and  $p = n + 1 = 12487$ , prime;
:   for  $p = 283$ ,  $n = 124818$  and  $p = n + 1 = 124819$ , prime;
:   for  $p = 307$ ,  $n = 18042$  and  $p = n + 1 = 18043$ , prime;
:   for  $p = 311$ ,  $n = 1866$  and  $p = n + 1 = 1867$ , prime;
:   for  $p = 347$ ,  $n = 182442$  and  $p = n + 1 = 182443$ , prime;
:   (...)
:   for  $p = 104677$ ,  $n = 6024364242$  and  $p = n + 1 = 6024364243$ , prime;
:   (...)
```

## 29. Two conjectures on the numbers obtained concatenating the squares of two primes $q$ and $r$ where $r = q + 18n$

**Abstract.** In this paper I state the following two conjectures: (I) For any prime  $q$  greater than 5 there exist an infinity of primes  $p$  obtained subtracting the square of  $q$  or the square of  $r$  from the number obtained concatenating the square of  $q$  with the square of  $r$ , where  $r$  prime,  $r = q + 18 \cdot n$ , and adding 1 (for example,  $p = 121841 - 121 + 1 = 121721$ , prime, also  $p = 121841 - 841 + 1 = 121001$ , prime, where  $q^2 = 11^2 = 121$ ,  $r^2 = 29^2 = 841$  and  $29 = 11 + 18 \cdot 1$ ); (II) For any positive integer  $n$  there exist an infinity of triplets of primes  $[p, q, r]$  such that  $r = q + 18 \cdot n$  and  $p$  is obtained subtracting the square of  $q$  or the square of  $r$  from the number obtained concatenating the square of  $q$  with the square of  $r$  and adding 1.

### Conjecture 1:

For any prime  $q$  greater than 5 there exist an infinity of primes  $p$  obtained subtracting the square of  $q$  or the square of  $r$  from the number obtained concatenating the square of  $q$  with the square of  $r$ , where  $r$  prime,  $r = q + 18 \cdot n$ , and adding 1.

Example:

:  $p = 121841 - 121 + 1 = 121721$ , prime, also  $p = 121841 - 841 + 1 = 121001$ , prime, where  $q^2 = 11^2 = 121$ ,  $r^2 = 29^2 = 841$  and  $29 = 11 + 18 \cdot 1$ .

Examples of primes  $p$  for  $q = 7$ :

:  $p = 490001 = 491849 - 1849 + 1$ , where  $[r, n] = [43, 2]$ ;  
 :  $p = 496193 = 496241 - 49 + 1$ , where  $[r, n] = [79, 4]$ ;  
 :  $p = 499361 = 499409 - 49 + 1$ , where  $[r, n] = [97, 5]$ ;  
 :  $p = 4900001 = 4922801 - 22801 + 1$ , where  $[r, n] = [151, 8]$ .

Examples of primes  $p$  for  $q = 11$ :

:  $p = 121721 = 121841 - 121 + 1$ , where  $[r, n] = [29, 1]$ ;  
 :  $p = 121001 = 121841 - 841 + 1$ , where  $[r, n] = [29, 1]$ ;  
 :  $p = 12136361 = 12136481 - 121 + 1$ , where  $[r, n] = [191, 10]$ .

Examples of primes  $p$  for  $q = 13, 17, 19, 23, 29, 31, 37$ :

:  $p = 16919153 = 16919321 - 169 + 1$ , where  $[q, r, n] = [13, 139, 7]$ ;  
 :  $p = 2890001 = 2892809 - 289 + 1$ , where  $[q, r, n] = [17, 53, 2]$ ;  
 :  $p = 3614969 = 3615329 - 361 + 1$ , where  $[q, r, n] = [19, 73, 3]$ ;  
 :  $p = 5292953 = 5293481 - 529 + 1$ , where  $[q, r, n] = [23, 59, 2]$ ;  
 :  $p = 8410001 = 8412209 - 2209 + 1$ , where  $[q, r, n] = [29, 47, 1]$ ;  
 :  $p = 96100001 = 96110609 - 10609 + 1$ , where  $[q, r, n] = [31, 103, 4]$ ;  
 :  $p = 13693961 = 13695329 - 1369 + 1$ , where  $[q, r, n] = [37, 73, 2]$ .

## Conjecture 2:

For any positive integer  $n$  there exist an infinity of triplets of primes  $[p, q, r]$  such that  $r = q + 18 \cdot n$  and  $p$  is obtained subtracting the square of  $q$  or the square of  $r$  from the number obtained concatenating the square of  $q$  with the square of  $r$  and adding 1.

Examples of primes  $p$  for  $n = 0$ :

- :  $p = 121001 = 121121 - 121 + 1$ , where  $r = q = 11$ ;
- :  $p = 289001 = 289289 - 288 + 1$ , where  $r = q = 17$ ;
- :  $p = 361001 = 361361 - 360 + 1$ , where  $r = q = 19$ .

Examples of primes  $p$  for  $n = 1$ :

- :  $p = 121721 = 121841 - 121 + 1$ , where  $[q, r] = [11, 29]$ ;
- :  $p = 121001 = 121841 - 841 + 1$ , where  $[q, r] = [11, 29]$ ;
- :  $p = 8410001 = 8412209 - 2208 + 1$ , where  $[q, r] = [29, 47]$ ;
- :  $p = 18490001 = 18493721 - 3721 + 1$ , where  $[q, r] = [43, 61]$ .

Examples of primes  $p$  for  $n = 2$ :

- :  $p = 490001 = 491849 - 1849 + 1$ , where  $[q, r] = [7, 43]$ ;
- :  $p = 8410001 = 8412209 - 2209 + 1$ , where  $[q, r] = [29, 47]$ ;
- :  $p = 13693961 = 13695329 - 1369 + 1$ , where  $[q, r] = [37, 73]$ ;
- :  $p = 18490001 = 18496241 - 6241 + 1$ , where  $[q, r] = [43, 79]$ .

Examples of primes  $p$  for  $n = 3$ :

- :  $p = 2894753 = 2895041 - 289 + 1$ , where  $[q, r] = [17, 71]$ ;
- :  $p = 2890001 = 2895041 - 5041 + 1$ , where  $[q, r] = [17, 71]$ .

Examples of primes  $p$  for  $n = 4, 5, 6, 7, 8, 9, 10$ :

- :  $p = 496193 = 496241 - 49 + 1$ , where  $[q, r, n] = [7, 79, 4]$ ;
- :  $p = 499361 = 499409 - 49 + 1$ , where  $[q, r, n] = [7, 97, 5]$ ;
- :  $p = 96100001 = 96119321 - 19321 + 1$ , where  $[q, r, n] = [31, 139, 6]$ ;
- :  $p = 16919153 = 16919321 - 169 + 1$ , where  $[q, r, n] = [13, 139, 7]$ ;
- :  $p = 4900001 = 4922801 - 22801 + 1$ , where  $[q, r, n] = [7, 151, 8]$ ;
- :  $p = 96136289 = 96137249 - 961 + 1$ , where  $[q, r, n] = [31, 193, 9]$ ;
- :  $p = 12136361 = 12136481 - 121 + 1$ , where  $[q, r, n] = [11, 191, 10]$ .

## Observation:

Because a pair  $[q, r]$  of emirps (reversible, but different, primes, see A006567 in OEIS) respects always the relation  $r = q + 18 \cdot n$ , there exist also primes  $p$  obtained as above; examples:

- :  $p = 2894753 = 2895041 - 289 + 1$ , where  $[q, r] = [17, 71]$ ;
- :  $p = 2890001 = 2895041 - 5041 + 1$ , where  $[q, r] = [17, 71]$ ;
- :  $p = 13693961 = 13695329 - 5041 + 1$ , where  $[q, r] = [37, 73]$ ;
- :  $p = 24649539353 = 24649564001 - 24649 + 1$ , where  $[q, r] = [157, 751]$ ;
- :  $p = 39601942481 = 39601982081 - 39600 + 1$ , where  $[q, r] = [199, 991]$ ;
- :  $p = 39601000001 = 39601982081 - 982081 + 1$ , where  $[q, r] = [199, 991]$ ;
- :  $p = 22201000001 = 22201885481 - 885480 + 1$ , where  $[q, r] = [149, 941]$ .

### 30. Eight formulas that generate semiprimes $pq$ such that $n = q - p + 1$ is prime respectively $n = p + q - 1$ is prime

**Abstract.** In this paper I list a number of eight formulas that generate two certain types of semiprimes, i.e. semiprimes  $p*q$  with the property that  $n = q - p + 1$  is prime respectively with the property that  $P + q - 1$  is prime.

#### Formulas that generate semiprimes $m = p*q$ such that $n = q - p + 1$ is prime

- (1)  $m = 30*x*y + 1$ , where  $y - x + 1$  is prime.

Examples:

- :  $m = 30*7*173 + 1$  ( $173 - 7 + 1 = 167$ , prime); it can be seen that  $m = 36331 = 47*773$  and  $n = 773 - 47 + 1 = 727$ , prime);
- :  $m = 30*7*263 + 1$  ( $263 - 7 + 1 = 257$ , prime); it can be seen that  $m = 55231 = 11*5021$  and  $n = 773 - 47 + 1 = 727$ , prime);
- :  $m = 30*7*269 + 1$  ( $269 - 7 + 1 = 263$ , prime); it can be seen that  $m = 56491 = 17*3323$  and  $n = 3323 - 17 + 1 = 3307$ , prime);
- :  $m = 30*7*509 + 1$  ( $509 - 7 + 1 = 503$ , prime); it can be seen that  $m = 106891 = 139*769$  and  $n = 769 - 139 + 1 = 631$ , prime).

- (2)  $m$  is the number obtained concatenating to the right a square of prime with the digit 1.

Examples:

- : for  $11^2 = 121$ ,  $m = 1211 = 7*173$  and  $n = 173 - 7 + 1 = 167$ , prime;
- : for  $13^2 = 169$ ,  $m = 1691 = 19*89$  and  $n = 89 - 19 + 1 = 11$ , prime;
- : for  $17^2 = 289$ ,  $m = 2891 = 49*59$  and  $n = 59 - 49 + 1 = 11$ , prime;
- : for  $31^2 = 961$ ,  $m = 9611 = 7*1373$  and  $n = 1373 - 7 + 1 = 1367$ , prime;
- : for  $53^2 = 2809$ ,  $m = 28091 = 7*4013$  and  $n = 4013 - 7 + 1 = 4007$ , prime;
- : for  $59^2 = 3481$ ,  $m = 34811 = 7*4973$  and  $n = 4973 - 7 + 1 = 4967$ , prime;
- : for  $67^2 = 3721$ ,  $m = 37211 = 127*293$  and  $n = 293 - 127 + 1 = 167$ , prime;
- : for  $79^2 = 6241$ ,  $m = 62411 = 139*449$  and  $n = 449 - 139 + 1 = 311$ , prime.

- (3)  $m$  is the product of the numbers obtained concatenating to the right a multiple of 3 with 1 respectively with 11.

Examples:

- :  $m = 31*311$  and  $n = 311 - 31 + 1 = 281$ , prime;
- :  $m = 151*1511$  and  $n = 1511 - 151 + 1 = 1361$ , prime;
- :  $m = 211*2111$  and  $n = 2111 - 211 + 1 = 1901$ , prime;
- :  $m = 271*2711$  and  $n = 2711 - 271 + 1 = 2441$ , prime;
- :  $m = 691*6911$  and  $n = 6911 - 691 + 1 = 6221$ , prime;
- :  $m = 1291*12911$  and  $n = 12911 - 1291 + 1 = 11621$ , prime;
- :  $m = 1831*18311$  and  $n = 18311 - 1831 + 1 = 16481$ , prime;
- :  $m = 2251*22511$  and  $n = 22511 - 2251 + 1 = 20261$ , prime.

- (4)  $m = (n + 1) \cdot p^2 - n$ , where  $p$  is prime and  $n$  positive integer.

Examples:

- : for  $n = 5$  the formula becomes  $m = 6 \cdot p^2 - 5$ :
  - : for  $p = 11$ ,  $m = 721 = 7 \cdot 103$  and  $n = 103 - 7 + 1 = 97$ , prime;
  - : for  $p = 41$ ,  $m = 10081 = 17 \cdot 593$  and  $n = 593 - 17 + 1 = 577$ , prime;
- : for  $n = 6$  the formula becomes  $m = 7 \cdot p^2 - 6$ :
  - : for  $p = 13$ ,  $m = 1177 = 11 \cdot 107$  and  $n = 107 - 11 + 1 = 97$ , prime;
  - : for  $p = 37$ ,  $m = 9577 = 61 \cdot 157$  and  $n = 157 - 61 + 1 = 97$ , prime;
  - : for  $p = 61$ ,  $m = 11761 = 19 \cdot 619$  and  $n = 619 - 19 + 1 = 601$ , prime;
- : for  $n = 7$  the formula becomes  $m = 8 \cdot p^2 - 7$ :
  - : for  $p = 17$ ,  $m = 2305 = 5 \cdot 461$  and  $n = 461 - 5 + 1 = 457$ , prime.

- (5)  $m$  is the number obtained concatenating two Poulet numbers.

Examples:

- : for  $P_1 = 561$  and  $P_2 = 1729$ ,  $m = 5611729 = 73 \cdot 76873$  and  $n = 76873 - 73 + 1 = 76801$ , prime;
- : for  $P_1 = 561$  and  $P_2 = 4033$ ,  $m = 5614033 = 643 \cdot 8731$  and  $n = 8731 - 643 + 1 = 8089$ ;
- : for  $P_1 = 4033$  and  $P_2 = 561$ ,  $m = 4033561 = 7 \cdot 576223$  and  $n = 576223 - 7 + 1 = 576217$ , prime;
- : for  $P_1 = 645$  and  $P_2 = 1729$ ,  $m = 6451729 = 571 \cdot 11299$  and  $n = 11299 - 571 + 1 = 10729$ , prime;
- : for  $P_1 = 645$  and  $P_2 = 4033$ ,  $m = 6454033 = 17 \cdot 379649$  and  $n = 379649 - 17 + 1 = 379633$ , prime.

- (6)  $m$  is the number obtained concatenating two squares of primes.

Examples:

- : for  $19^2 = 361$  and  $11^2 = 121$ ,  $m = 361121 = 331 \cdot 1091$  and  $n = 1091 - 331 + 1 = 761$ , prime;
- : for  $11^2 = 361$  and  $47^2 = 2209$ ,  $m = 1212209 = 97 \cdot 12497$  and  $n = 12497 - 97 + 1 = 12401$ , prime;
- : for  $41^2 = 1681$  and  $31^2 = 961$ ,  $m = 1681961 = 367 \cdot 4583$  and  $n = 4583 - 367 + 1 = 4217$ , prime.

- 7)  $m$  is the number  $1abc$  obtained concatenating 1 with  $a$ , prime, then with  $b$  prime such that  $b = a + 6$  and then with  $c$  prime such that  $c = b + 6$ .

Examples:

- : for  $[a, b, c] = [31, 37, 43]$ ,  $m = 1313743 = 17 \cdot 77279$  and  $m = 77279 - 17 + 1 = 77263$ , prime;
- : for  $[a, b, c] = [47, 53, 59]$ ,  $m = 1475359 = 127 \cdot 11617$  and  $m = 11617 - 127 + 1 = 11491$ , prime;
- : for  $[a, b, c] = [97, 103, 109]$ ,  $m = 197103109 = 7 \cdot 28157587$  and  $m = 28157587 - 7 + 1 = 28157581$ , prime.

## Formulas that generate semiprimes $m = p \cdot q$ such that $n = P + q + 1$ is prime

- (1)  $m = 30 \cdot x \cdot y - 1$ , where  $x + y - 1$  is prime.

Examples:

- :  $m = 30 \cdot 7 \cdot 257 - 1$  ( $257 + 7 - 1 = 263$ , prime); it can be seen that  $m = 53969 = 29 \cdot 1861$  and  $n = 29 + 1861 - 1 = 1889$ , prime);
- :  $m = 30 \cdot 7 \cdot 433 - 1$  ( $433 + 7 - 1 = 439$ , prime); it can be seen that  $m = 90929 = 79 \cdot 1151$  and  $n = 79 + 1151 - 1 = 1229$ , prime);
- :  $m = 30 \cdot 7 \cdot 563 - 1$  ( $563 + 7 - 1 = 569$ , prime); it can be seen that  $m = 118229 = 191 \cdot 619$  and  $n = 191 + 619 - 1 = 809$ , prime);
- :  $m = 30 \cdot 7 \cdot 571 - 1$  ( $571 + 7 - 1 = 577$ , prime); it can be seen that  $m = 119909 = 19 \cdot 6311$  and  $n = 19 + 6311 - 1 = 6329$ , prime).

- (2)  $m$  is the number obtained concatenating to the right a square of prime with the digit 1.

Examples:

- : for  $11^2 = 121$ ,  $m = 1211 = 7 \cdot 173$  and  $n = 173 + 7 - 1 = 179$ , prime;
- : for  $13^2 = 169$ ,  $m = 1691 = 19 \cdot 89$  and  $n = 89 + 19 - 1 = 109$ , prime;
- : for  $17^2 = 289$ ,  $m = 2891 = 49 \cdot 59$  and  $n = 59 + 49 - 1 = 109$ , prime;
- : for  $19^2 = 361$ ,  $m = 3611 = 23 \cdot 157$  and  $n = 157 + 23 - 1 = 179$ , prime;
- : for  $29^2 = 841$ ,  $m = 8411 = 13 \cdot 647$  and  $n = 647 + 13 - 1 = 659$ , prime;
- : for  $53^2 = 2809$ ,  $m = 28091 = 7 \cdot 4013$  and  $n = 4013 + 7 - 1 = 4019$ , prime;
- : for  $67^2 = 3721$ ,  $m = 37211 = 127 \cdot 293$  and  $n = 293 + 127 - 1 = 419$ , prime;
- : for  $79^2 = 6241$ ,  $m = 62411 = 139 \cdot 449$  and  $n = 449 + 139 - 1 = 587$ , prime.

- (3)  $m = (n + 1) \cdot p^2 - n$ , where  $p$  is prime and  $n$  positive integer.

Examples:

- : for  $n = 6$  the formula becomes  $m = 6 \cdot p^2 - 5$ :
  - : for  $p = 11$ ,  $m = 721 = 7 \cdot 103$  and  $n = 103 + 7 - 1 = 109$ , prime;
  - : for  $p = 31$ ,  $m = 5761 = 7 \cdot 823$  and  $n = 823 + 7 - 1 = 829$ , prime;
- : for  $n = 8$  the formula becomes  $m = 8 \cdot p^2 - 7$ :
  - : for  $p = 19$ ,  $m = 2881 = 43 \cdot 67$  and  $n = 67 + 43 - 1 = 109$ , prime.

- (4)  $m$  is the number obtained concatenating two Poulet numbers.

Examples:

- : for  $P1 = 1905$  and  $P2 = 341$ ,  $m = 1905341 = 251 \cdot 7591$  and  $n = 7591 + 251 - 1 = 7841$ , prime;
- : for  $P1 = 561$  and  $P2 = 1387$ ,  $m = 5611387 = 337 \cdot 16651$  and  $n = 16651 + 337 - 1 = 16987$ , prime;
- : for  $P1 = 2701$  and  $P2 = 561$ ,  $m = 2701561 = 43 \cdot 62827$  and  $n = 62827 + 43 - 1 = 62869$ , prime;
- : for  $P1 = 2465$  and  $P2 = 341$ ,  $m = 2465341 = 1237 \cdot 1993$  and  $n = 1993 + 1237 - 1 = 3229$ , prime.

- (5)  $m$  is the number obtained concatenating two squares of primes.

Examples:

: for  $13^2 = 169$  and  $7^2 = 49$ ,  $m = 16949 = 17 \cdot 997$  and  $n = 997 + 17 - 1 = 1013$ , prime;  
: for  $7^2 = 49$  and  $31^2 = 961$ ,  $m = 49961 = 47 \cdot 1063$  and  $n = 1063 + 47 - 1 = 1109$ , prime;  
: for  $11^2 = 121$  and  $41^2 = 1681$ ,  $m = 1211861 = 709 \cdot 1709$  and  $n = 1709 + 709 - 1 = 2417$ , prime;  
: for  $13^2 = 169$  and  $31^2 = 961$ ,  $m = 169961 = 11 \cdot 15451$  and  $n = 15451 + 11 - 1 = 15461$ , prime;  
: for  $23^2 = 289$  and  $53^2 = 2809$ ,  $m = 2892809 = 1217 \cdot 2377$  and  $n = 1217 + 2377 - 1 = 3593$ , prime;  
: for  $43^2 = 1849$  and  $19^2 = 361$ ,  $m = 1849361 = 23 \cdot 80407$  and  $n = 23 + 80407 - 1 = 80429$ , prime;  
: for  $29^2 = 841$  and  $31^2 = 961$ ,  $m = 841961 = 23 \cdot 36607$  and  $n = 23 + 36607 - 1 = 36629$ , prime;  
: for  $43^2 = 1849$  and  $29^2 = 841$ ,  $m = 1849841 = 7 \cdot 264263$  and  $n = 7 + 264263 - 1 = 264269$ , prime;  
: for  $43^2 = 1849$  and  $31^2 = 961$ ,  $m = 1849961 = 41 \cdot 45121$  and  $n = 41 + 45121 - 1 = 45161$ , prime;  
: for  $37^2 = 1369$  and  $43^2 = 1849$ ,  $m = 13691849 = 89 \cdot 153841$  and  $n = 89 + 153841 - 1 = 153929$ , prime.

- (6)  $m$  is a number formed from one digit or a group of digits concatenated with itself an odd number of times then to the left and to the right with digit 1.

Examples:

:  $m = 133333331 = 11287 \cdot 11813$  and  $11287 + 11813 - 1 = 23099$ , prime;  
:  $m = 14441 = 7 \cdot 2063$  and  $7 + 2063 - 1 = 2069$ , prime;  
:  $m = 1555555551 = 1709 \cdot 9102139$  and  $1709 + 9102139 - 1 = 9103847$ , prime;  
:  $m = 13434341 = 373 \cdot 36017$  and  $373 + 36017 - 1 = 36389$ , prime;  
:  $m = 14343431 = 59 \cdot 243109$  and  $59 + 243109 - 1 = 243167$ , prime;  
:  $m = 13535351 = 61 \cdot 221891$  and  $61 + 221891 - 1 = 221951$ , prime;  
:  $m = 17676761 = 3529 \cdot 5009$  and  $5009 + 3529 - 1 = 8537$ , prime;  
:  $m = 18989891 = 131 \cdot 144961$  and  $131 + 144961 - 1 = 145091$ , prime;  
:  $m = 13343343341 = 20047 \cdot 665603$  and  $20047 + 665603 - 1 = 685649$ , prime.

## 31. Seven Smarandache-Coman sequences of primes

**Abstract.** In a previous paper, “Fourteen Smarandache-Coman sequences of primes”, I defined the “Smarandache-Coman sequences” as “all the sequences of primes obtained from the Smarandache concatenated sequences using basic arithmetical operations between the terms of such a sequence, like for instance the sum or the difference between two consecutive terms plus or minus a fixed positive integer, the partial sums, any other possible basic operations between terms like  $a(n) + a(n+2) - a(n+1)$ , or on a term like  $a(n) + S(a(n))$ , where  $S(a(n))$  is the sum of the digits of the term  $a(n)$  etc.” In this paper I extend the notion to the sequences of primes obtained from the Smarandache concatenated sequences using any arithmetical operation and I present seven sequences obtained from the Smarandache concatenated sequences using concatenation between the terms of the sequence and other numbers and also fourteen conjectures on them.

### Introduction:

In a previous paper, “Fourteen Smarandache-Coman sequences of primes”, I defined the “Smarandache-Coman sequences” as “all the sequences of primes obtained from the Smarandache concatenated sequences using basic arithmetical operations between the terms of such a sequence, like for instance the sum or the difference between two consecutive terms plus or minus a fixed positive integer, the partial sums, any other possible basic operations between terms like  $a(n) + a(n+2) - a(n+1)$ , or on a term like  $a(n) + S(a(n))$ , where  $S(a(n))$  is the sum of the digits of the term  $a(n)$  etc.” In this paper I extend the notion to the sequences of primes obtained from the Smarandache concatenated sequences using any arithmetical operation and I present seven sequences obtained from the Smarandache concatenated sequences using concatenation between the terms of the sequence and other numbers and also fourteen conjectures on them.

Note: The Smarandache concatenated sequences are well known for the very few terms which are primes; on the contrary, many Smarandache-Coman sequences can be constructed that probably have an infinity of terms (primes, by definition).

Note: I shall use the notation  $a(n)$  for a term of a Smarandache concatenated sequence and  $b(n)$  for a term of a Smarandache-Coman sequence.

### SEQUENCE I

Starting from the Smarandache consecutive numbers sequence (defined as the sequence obtained through the concatenation of the first  $n$  positive integers, see A007908 in OEIS), we define the following Smarandache-Coman sequence:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

We have:

- :  $b(1) = 11$ , prime;
- :  $b(3) = 1231$ , prime;
- :  $b(9) = 1234567891$ , prime;

:  $b(11) = 12345678910111$ , prime;  
 :  $b(16) = 123456789101112131415161$ , prime;  
 :  $b(26) = 12345678910111213141516171819202122232425261$ , prime;  
 (...)

I also conjecture that there exist an infinity of terms  $b(n)$  which are semiprimes (some of them  $p \cdot q$  having the interesting property that  $q - p + 1$  is prime; such terms are:  $b(5) = 123451 = 41 \cdot 3011$  and  $3011 - 41 + 1 = 2971$ ;  $b(6) = 1234561 = 211 \cdot 5851$  and  $5851 - 211 + 1 = 5641$ , prime).

## SEQUENCE II

Starting from the Smarandache concatenated odd sequence (defined as the sequence obtained through the concatenation of the odd numbers from 1 to  $2 \cdot n - 1$ , see A019519 in OEIS), we define the following Smarandache-Coman sequence:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

We have:

:  $b(1) = 11$ , prime;  
 :  $b(2) = 131$ , prime;  
 :  $b(9) = 13579111315171$ , prime;  
 :  $b(10) = 1357911131517191$ , prime;  
 :  $b(12) = 13579111315171921231$ , prime;  
 :  $b(15) = 13579111315171921232527291$ , prime;  
 (...)

I also conjecture that there exist an infinity of terms  $b(n)$  which are semiprimes.

## SEQUENCE III

Starting from the Smarandache reverse sequence (defined as the sequence obtained through the concatenation of the first  $n$  positive integers in reverse order, see A000422 in OEIS), we define the following Smarandache-Coman sequence:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

We have:

:  $b(1) = 11$ , prime;  
 :  $b(2) = 211$ , prime;  
 :  $b(8) = 876543211$ , prime;  
 :  $b(9) = 9876543211$ , prime;  
 :  $b(22) = 222120191817161514131211109876543211$ , prime;  
 :  $b(26) = 12345678910111213141516171819202122232425261$ , prime;  
 (...)

I also conjecture that there exist an infinity of terms  $b(n)$  which are semiprimes, some of them having the interesting property that one of the factor is much larger than the other one; such terms are:

:  $b(15) = 1514131211109876543211 = 29 * 52211421072754363559$ ;  
 :  $b(17) = 17161514131211109876543211 = 359 * 47803660532621475979229$ ;  
 :  $b(18) = 1817161514131211109876543211 = 31 * 58618113359071326125049781$ ;  
 :  $b(31) = 313029282726252423222120191817161514131211109876543211 = 519373 * 602706114346052688957878426135285265370381421207$ .

#### SEQUENCE IV

Starting from the Smarandache  $n2*n$  sequence (the  $n$ -th term of the sequence is obtained concatenating the numbers  $n$  and  $2*n$ , see A019550 in OEIS), we define the following Smarandache-Coman sequence:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

We have:

:  $b(2) = 241$ , prime;  
 :  $b(5) = 5101$ , prime;  
 :  $b(6) = 6121$ , prime;  
 :  $b(8) = 8161$ , prime;  
 :  $b(9) = 9181$ , prime;  
 :  $b(12) = 12241$ , prime;  
 :  $b(14) = 14281$ , prime;  
 :  $b(17) = 17341$ , prime;  
 :  $b(19) = 19381$ , prime;  
 :  $b(22) = 22441$ , prime;  
 :  $b(24) = 24481$ , prime;  
 (...)  
 :  $b(104) = 1042081$ , prime;  
 :  $b(106) = 1062121$ , prime;  
 :  $b(108) = 1082161$ , prime;  
 :  $b(110) = 1102201$ , prime;  
 :  $b(112) = 1122241$ , prime;  
 (...)  
 :  $b(1004) = 100420081$ , prime;  
 :  $b(1007) = 100720141$ , prime;  
 :  $b(1011) = 101120221$ , prime;  
 (...)

I also conjecture that there exist an infinity of terms  $b(n)$  which are semiprimes, as well as an infinity of terms  $b(n)$  which are squares of primes: such terms are  $b(1) = 121 = 11^2$ ,  $b(3) = 361 = 19^2$ ,  $b(10) = 10201 = 101^2$ .

#### SEQUENCE V

Starting again from the Smarandache  $n2*n$  sequence (the  $n$ -th term of the sequence is obtained concatenating the numbers  $n$  and  $2*n$ , see A019550 in OEIS), we define the following Smarandache-Coman sequence:  $b(n) = 1a(n)1$ , i.e. the terms of the Smarandache sequence concatenated both to the left and to the right with the number 1. I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

We have:

```

:      b(3) = 1361, prime;
:      b(4) = 1481, prime;
:      b(5) = 15101, prime;
:      b(9) = 19181, prime;
:      b(12) = 112241, prime;
:      b(14) = 114281, prime;
:      b(15) = 115301, prime;
:      b(18) = 118361, prime;
:      b(20) = 120401, prime;
:      b(21) = 121421, prime;
:      (...)
:      b(100) = 11002001, prime;
:      b(104) = 11042081, prime;
:      b(105) = 11052101, prime;
:      b(107) = 11072141, prime;
:      b(108) = 11082161, prime;
:      (...)

```

I also conjecture that there exist an infinity of terms  $b(n)$  which are semiprimes.

#### SEQUENCE VI

Starting from the Smarandache  $nn^2$  sequence (the  $n$ -th term of the sequence is obtained concatenating the numbers  $n$  and  $n^2$ , see A053061 in OEIS), we define the following Smarandache-Coman sequence:  $b(n) = a(n)1$ , i.e. the terms of the Smarandache sequence concatenated to the right with the number 1. I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

We have:

```

:      b(2) = 241, prime;
:      b(6) = 6361, prime;
:      b(8) = 8641, prime;
:      b(9) = 9181, prime;
:      b(11) = 111211, prime;
:      b(12) = 121441, prime;
:      b(29) = 298411, prime;
:      (...)

```

I also conjecture that there exist an infinity of terms  $b(n)$  which are semiprimes.

#### SEQUENCE VII

Starting again from the Smarandache  $nn^2$  sequence (the  $n$ -th term of the sequence is obtained concatenating the numbers  $n$  and  $n^2$ , see A053061 in OEIS), we define the following Smarandache-Coman sequence:  $b(n) = 1a(n)1$ , i.e. the terms of the Smarandache sequence concatenated both to the left and to the right with the number 1. I conjecture that there exist an infinity of terms  $b(n)$  which are primes.

We have:

:  $b(6) = 16361$ , prime;  
:  $b(7) = 17491$ , prime;  
:  $b(11) = 111211$ , prime;  
:  $b(18) = 1183241$ , prime;  
:  $b(26) = 1266761$ , prime;  
:  $b(28) = 1287841$ , prime;  
(...)

I also conjecture that there exist an infinity of terms  $b(n)$  which are semiprimes.

## 32. Two conjectures on Smarandache's divisor products sequence

**Abstract.** In this paper I make the following two conjectures on the *Smarandache's divisor products sequence* where a term  $P(n)$  of the sequence is defined as the product of the positive divisors of  $n$ : (1) there exist an infinity of  $n$  composites such that the number  $m = P(n) + n - 1$  is prime; (2) there exist an infinity of  $n$  composites such that the number  $m = P(n) - n + 1$  is prime.

The *Smarandache's divisor products sequence* (see A007955 in OEIS):

: 1, 2, 3, 8, 5, 36, 7, 64, 27, 100, 11, 1728, 13, 196, 225, 1024, 17, 5832, 19, 8000, 441, 484, 23, 331776, 125, 676, 729, 21952, 29, 810000, 31, 32768, 1089, 1156, 1225, 10077696, 37, 1444, 1521, 2560000, 41, 3111696, 43, 85184, 91125, 2116, 47, 254803968 (...)

### Conjecture 1:

Let  $P(n)$  be the *Smarandache's divisor products sequence* where a term  $P(n)$  of the sequence is defined as the product of the positive divisors of  $n$ : there exist an infinity of  $n$  composites such that the number  $m = P(n) + n - 1$  is prime.

Note that for  $n$  primes, because  $P(n) = n$ ,  $P(n) + n - 1 = 2*n - 1$  and is already conjectured that there exist an infinity of primes of the form  $2*q - 1$ , where  $q$  prime.

The sequence of primes  $m$ :

:  $m = 3$ , prime, for  $(n, P(n)) = (2, 2)$ ;  
:  $m = 11$ , prime, for  $(n, P(n)) = (4, 8)$ ;  
:  $m = 41$ , prime, for  $(n, P(n)) = (6, 36)$ ;  
:  $m = 71$ , prime, for  $(n, P(n)) = (8, 64)$ ;  
:  $m = 109$ , prime, for  $(n, P(n)) = (10, 100)$ ;  
:  $m = 1739$ , prime, for  $(n, P(n)) = (12, 1728)$ ;  
:  $m = 239$ , prime, for  $(n, P(n)) = (15, 225)$ ;  
:  $m = 1039$ , prime, for  $(n, P(n)) = (16, 1024)$ ;  
:  $m = 5849$ , prime, for  $(n, P(n)) = (18, 5832)$ ;  
:  $m = 461$ , prime, for  $(n, P(n)) = (21, 441)$ ;  
:  $m = 149$ , prime, for  $(n, P(n)) = (25, 125)$ ;  
:  $m = 701$ , prime, for  $(n, P(n)) = (26, 676)$ ;  
:  $m = 1259$ , prime, for  $(n, P(n)) = (35, 1225)$ ;  
:  $m = 1481$ , prime, for  $(n, P(n)) = (38, 1444)$ ;  
:  $m = 2560039$ , prime, for  $(n, P(n)) = (40, 2560000)$ ;  
:  $m = 2161$ , prime, for  $(n, P(n)) = (46, 2116)$ ;  
(...)

Examples of larger  $m$ :

:  $m = 46656000059$ , prime, for  $(n, P(n)) = (60, 46656000000)$ ;  
:  $m = 782757789791$ , prime, for  $(n, P(n)) = (96, 782757789696)$ ;  
:  $m = 1586874323051$ , prime, for  $(n, P(n)) = (108, 1586874322944)$ ;  
:  $m = 634562281237119143$ , prime, for  $(n, P(n)) = (168, 634562281237118976)$ .

Note that  $m$  is prime for  $n = 12, 60, 96, 108, 168$ . I conjecture that  $m$  is prime for an infinity of  $n$  of the form  $12 \cdot k$ .

### Conjecture 2:

Let  $P(n)$  be the *Smarandache's divisor products sequence* where a term  $P(n)$  of the sequence is defined as the product of the positive divisors of  $n$ : there exist an infinity of  $n$  composites such that the number  $m = P(n) - n + 1$  is prime.

Note that for  $n$  primes, because  $P(n) = n$ ,  $P(n) - n + 1 = 1$ .

The sequence of primes  $m$ :

- :  $m = 5$ , prime, for  $(n, P(n)) = (4, 8)$ ;
- :  $m = 31$ , prime, for  $(n, P(n)) = (6, 36)$ ;
- :  $m = 19$ , prime, for  $(n, P(n)) = (9, 27)$ ;
- :  $m = 211$ , prime, for  $(n, P(n)) = (15, 225)$ ;
- :  $m = 1009$ , prime, for  $(n, P(n)) = (16, 1024)$ ;
- :  $m = 421$ , prime, for  $(n, P(n)) = (21, 441)$ ;
- :  $m = 463$ , prime, for  $(n, P(n)) = (22, 484)$ ;
- :  $m = 331753$ , prime, for  $(n, P(n)) = (24, 331776)$ ;
- :  $m = 149$ , prime, for  $(n, P(n)) = (25, 125)$ ;
- :  $m = 1123$ , prime, for  $(n, P(n)) = (34, 1156)$ ;
- :  $m = 254803921$ , prime, for  $(n, P(n)) = (48, 254803968)$ ;
- (...)

Examples of larger  $m$ :

- :  $m = 531440999911$ , prime, for  $(n, P(n)) = (90, 531441000000)$ ;
- :  $m = 389328928561$ , prime, for  $(n, P(n)) = (208, 389328928768)$ .

Note that  $m$  is prime for  $n = 24, 48$ . I conjecture that  $m$  is prime for an infinity of  $n$  of the form  $12 \cdot k$ .

## Part Two.

### Sequences of Fermat pseudoprimes and conjectures on them

#### 1. Generic form for a probably infinite sequence of Poulet numbers *i.e.* $2n^2 + 147n + 2701$

**Abstract.** In this paper I observe that the formula  $2n^2 + 147n + 2701$  produces Poulet numbers, and I conjecture that this formula is generic for an infinite sequence of Poulet numbers.

**The sequence of Poulet numbers of the form  $2n^2 + 147n + 2701$ :**

: 2701, 4371, 8911, 10585, 18721, 33153, 49141, 93961 (...)

These numbers were obtained for the following values of n:

: 0, 10, 30, 36, 60, 92, 120, 180 (...)

**Conjecture:**

There are infinite many Poulet numbers P of the form  $2n^2 + 147n + 2701$  (see A214016 posted by me on OEIS for a subsequence of the sequence from above, i.e. Poulet numbers of the form  $7200n^2 + 8820n + 2701$ ).

**Observation:**

Note the following interesting facts:

- : for  $P = 2701 = 37 \cdot 73$  both  $37 (= 2 \cdot 17 + 3)$  and  $73 (= 4 \cdot 17 + 5)$  can be written as  $17 \cdot m + m + 1$ , where m positive integer;
- : for  $p = 10585 = 5 \cdot 29 \cdot 73$  both  $5 \cdot 29 = 145 (= 8 \cdot 17 + 9)$  and  $73 (= 4 \cdot 17 + 5)$  can be written as  $17 \cdot m + m + 1$ ;
- : for  $p = 93961 = 7 \cdot 31 \cdot 433$  both  $7 \cdot 31 = 217 (= 12 \cdot 17 + 13)$  and  $433 (= 24 \cdot 17 + 25)$  can be written as  $17 \cdot m + m + 1$ .
- : for  $P = 4371 = 3 \cdot 31 \cdot 47$  both  $31 (= 2 \cdot 17 - 3)$  and  $47 (= 3 \cdot 17 - 4)$  can be written as  $17 \cdot m - m - 1$ , where m positive integer;
- : for  $P = 18721 = 97 \cdot 193$  both  $97 (= 6 \cdot 17 - 5)$  and  $193 (= 12 \cdot 17 - 11)$  can be written as  $17 \cdot m - m - 1$ ;
- : for  $p = 33153 = 3 \cdot 43 \cdot 257$  both  $3 \cdot 43 = 129 (= 8 \cdot 17 - 7)$  and  $257 (= 16 \cdot 17 - 15)$  can be written as  $17 \cdot m - m - 1$ .

Note the following subsequence of the sequence from above, obtained for  $n = 10 \cdot m$ :

: 2701, 4371, 8911, 18721, 49141, 93961, 226801, 314821, 534061, 665281, 915981 (...)

obtained for  $m = 0, 1, 3, 6, 12, 18, 30, 36, 48, 54, 64$  (...)

## 2. Generic form for a probably infinite sequence of Poulet numbers *i.e.* $4n^2 + 37n + 85$

**Abstract.** In this paper I observe that the formula  $4n^2 + 37n + 85$  produces Poulet numbers, and I conjecture that this formula is generic for an infinite sequence of Poulet numbers.

**The sequence of Poulet numbers of the form  $4n^2 + 37n + 85$ :**

: 1105, 1387, 2047, 3277, 6601, 13747, 16705, 19951, 31417, 74665, 83665, 88357, 90751 (...)

These numbers were obtained for the following values of  $n$ :

: 12, 14, 18, 24, 36, 54, 60, 66, 88, 132, 140, 144, 146 (...)

### Conjecture:

There are infinite many Poulet numbers of the form  $4n^2 + 37n + 85$  (see A214017 posted by me on OEIS for a subsequence of the sequence from above, *i.e.* Poulet numbers of the form  $144n^2 + 122n + 85$ ).

### Observation:

Note that almost all from the first 13 numbers  $P$  from the sequence above have a prime factor  $q$  of one from the following five forms:

(A)  $q = 17$  (for  $P = 1105 = 5 \cdot 13 \cdot 17$ );

(B)  $q$  is of the form  $17m + m + 1$  ( $q = 73 = 4 \cdot 17 + 5$  for  $P = 1387$ ,  $q = 109 = 6 \cdot 17 + 7$  for  $P = 74665$ );

(C)  $q$  is of the form  $17m + m - 1$  ( $q = 89 = 5 \cdot 17 + 4$  for  $P = 2047$  and  $P = 31417$ ;  $q = 233 = 13 \cdot 17 + 12$  for  $P = 13747$ ,  $q = 71 = 4 \cdot 17 + 3$  for  $P = 19951$ );

(D)  $q$  is of the form  $17m - m + 1$  ( $q = 113 = 7 \cdot 17 - 6$  for  $P = 3277$ ;  $q = 257 = 16 \cdot 17 - 15$  for  $P = 13747$ ;  $q = 353 = 22 \cdot 17 - 21$  for  $P = 31417$ ,  $q = 577 = 36 \cdot 17 - 35$  for  $P = 83665$ ,  $q = 593 = 37 \cdot 17 - 36$  for  $P = 88357$ );

(E)  $q$  is of the form  $17m - m - 1$ .

Exceptions:

:  $6601 = 7 \cdot 23 \cdot 41$ ; but, even in this case,  $7 \cdot 23 = 161 = 9 \cdot 17 + 8$  (case C),  $7 \cdot 41 = 16 \cdot 17 + 15$  (case C),  $23 \cdot 41 = 59 \cdot 17 - 60$  (case E);

:  $90751 = 151 \cdot 601$ ; but, even in this case,  $151 \cdot 601 = 5672 \cdot 17 - 5673$  (case E).

### 3. Two conjectures on Poulet numbers of the form $mn^2 + 11mn - 23n + 19m - 49$

**Abstract.** In this paper I observe that the formula  $m \cdot n^2 + 11 \cdot m \cdot n - 23 \cdot n + 19 \cdot m - 49$  produces Poulet numbers, and I conjecture that this formula produces an infinite sequence of Poulet numbers for any  $m$  non-null positive integer, respectively for any  $n$  non-null positive integer.

#### Conjecture 1:

The formula  $m \cdot n^2 + 11 \cdot m \cdot n - 23 \cdot n + 19 \cdot m - 49$  produces an infinite sequence of Poulet numbers for any  $n$  non-null positive integer.

##### Examples:

Formula becomes  $31 \cdot m - 72$  for  $n = 1$  and we have the following sequence of Poulet numbers  $P = 31 \cdot m - 72$  (obtained for  $m = 259, 367, 5111$ ):  
: 7957, 11305, 158369 (...)

Formula becomes  $45 \cdot m - 95$  for  $n = 2$  and we have the following sequence of Poulet numbers  $P = 45 \cdot m - 95$  (obtained for  $m = 888, 928, 2384$ ):  
: 39865, 41665, 107185(...)

Formula becomes  $61 \cdot m - 118$  for  $n = 3$  and we have the following sequence of Poulet numbers  $P = 61 \cdot m - 118$  (obtained for  $m = 329, 379$ ):  
: 19951, 23001(...)

Formula becomes  $99 \cdot m - 164$  for  $n = 5$  and we have the following sequence of Poulet numbers  $P = 99 \cdot m - 164$  (obtained for  $m = 319, 659, 1387$ ):  
: 31417, 65077, 137149(...)

#### Conjecture 2:

The formula  $m \cdot n^2 + 11 \cdot m \cdot n - 23 \cdot n + 19 \cdot m - 49$  produces an infinite sequence of Poulet numbers for any  $m$  non-null positive integer.

##### Examples:

Formula becomes  $3 \cdot n^2 + 10 \cdot n + 8$  for  $m = 3$  and we have the following sequence of Poulet numbers  $P = 3 \cdot n^2 + 10 \cdot n + 8$  (obtained for  $n = 9, 13, 27, 29, 35, 41, 51, 71, 91, 101, 149, 165$ ):  
: 341, 645, 2465, 2821, 4033, 5461, 8321, 15841, 25761, 31621, 68101, 83333 (...)

Formula becomes  $4 \cdot n^2 + 21 \cdot n + 27$  for  $m = 4$  and we have the following sequence of Poulet numbers  $P = 4 \cdot n^2 + 21 \cdot n + 27$  (obtained for  $n = 14, 16, 20, 26, 38, 56, 62, 68, 86, 134, 142, 146, 148$ ):  
: 1105, 1387, 2047, 3277, 6601, 13747, 16705, 19951, 31417, 83665, 88357, 90751 (...)

#### 4. Three cubic polynomials that generate sequences of Poulet numbers

**Abstract.** In this paper I present three cubic polynomials that generate (probably infinite) sequences of Poulet numbers.

##### I.

**Poulet numbers of the form  $240*n^3 - 2708*n^2 + 10172*n - 12719$ :**

: 340561, 2299081, 4335241, 8041345, 32085041, 153927961, 321524281 (...)

These numbers were obtained for the following values of n:

: 15, 25, 30, 36, 55, 90, 114 (...)

##### **Conjecture:**

There are infinite many Poulet numbers of the form  $240*n^3 - 2708*n^2 + 10172*n - 12719$  (see A182132 posted by me on OEIS for a subsequence of the sequence from above, i.e. Carmichael numbers of the form  $(30*n - 7)*(90*n - 23)*(300*n - 79)$ ).

##### II.

**Poulet numbers of the form  $80*n^3 + 788*n^2 + 2584*n + 2821$ :**

: 2821, 63973, 285541, 488881, 7428421(...)

These numbers were obtained for the following values of n:

: 0, 6, 12, 15, 42 (...)

##### **Conjecture:**

There are infinite many Poulet numbers of the form  $80*n^3 + 788*n^2 + 2584*n + 2821$  (see A182085 posted by me on OEIS for a subsequence of the sequence from above, i.e. Carmichael numbers of the form  $(30*n + 7)*(60*n + 13)*(150*n + 31)$ ). 2

##### III

**Poulet numbers of the form  $120*n^3 - 3148*n^2 + 27522*n - 80189$ :**

: 29341, 1152271, 11875821, 16158331, 34901461 (...)

These numbers were obtained for the following values of n:

: 15, 30, 55, 60, 75 (...)

##### **Conjecture:**

There are infinite many Poulet numbers of the form  $120*n^3 - 3148*n^2 + 27522*n - 80189$  (see A182133 posted by me on OEIS for a subsequence of the sequence from above, i.e. Carmichael numbers of the form  $(30*n - 17)*(90*n - 53)*(150*n - 89)$ ).

## 5. Conjecture on Poulet numbers of the form $8mn^3 + 40n^3 + 38n^2 + 6mn^2 + mn + 11n + 1$

**Abstract.** In this paper I observe that the formula  $8m^*n^3 + 40n^3 + 38n^2 + 6m^*n^2 + m^*n + 11n + 1$  produces Poulet numbers, and I conjecture that this formula produces an infinite sequence of Poulet numbers for any  $m$  non-null positive integer.

### Conjecture:

The formula  $8m^*n^3 + 40n^3 + 38n^2 + 6m^*n^2 + m^*n + 11n + 1$  produces an infinite sequence of Poulet numbers for any  $m$  non-null positive integer.

### Examples:

Formula becomes  $48n^3 + 44n^2 + 12n + 1$  for  $m = 1$  and we have the following sequence of Poulet numbers  $P = 48n^3 + 44n^2 + 12n + 1$  (obtained for  $n = 3, 7, 15, 18, 33, 45, 66 \dots$ ):

: 1729, 18705, 172081, 294409, 1773289, 4463641, 13992265 (...)

Formula becomes  $56n^3 + 50n^2 + 13n + 1$  for  $m = 2$  and we have the following sequence of Poulet numbers  $P = 56n^3 + 50n^2 + 13n + 1$  (obtained for  $n = 64, \dots$ ):

: 14885697 (...)

Formula becomes  $64n^3 + 56n^2 + 14n + 1$  for  $m = 3$  and we have the following sequence of Poulet numbers  $P = 64n^3 + 56n^2 + 14n + 1$  (obtained for  $n = 44, \dots$ ):

: 5560809 (...)

Formula becomes  $80n^3 + 68n^2 + 16n + 1$  for  $m = 5$  and we have the following sequence of Poulet numbers  $P = 80n^3 + 68n^2 + 16n + 1$  (obtained for  $n = 3, 9, 15, 18, 45 \dots$ ):

: 2821, 63973, 285541, 488881, 7428421 (...)

Note that all the solutions obtained for  $n$  so far (up to  $n = 45$ ) are of the form  $3k$ .

Formula becomes  $112n^3 + 92n^2 + 20n + 1$  for  $m = 9$  and we have the following sequence of Poulet numbers  $P = 112n^3 + 92n^2 + 20n + 1$  (obtained for  $n = 15, 45, \dots$ ):

: 399001, 10393201 (...)

## 6. Conjecture on Poulet numbers of the form $9mn^3 + 3n^3 - 15mn^2 + 6mn - 2n^2$

**Abstract.** In this paper I observe that the formula  $9m^*n^3 + 3n^3 - 15m^*n^2 + 6m^*n - 2n^2$  produces Poulet numbers, and I conjecture that this formula produces an infinite sequence of Poulet numbers for any  $m$  non-null positive integer.

### Conjecture:

The formula  $9m^*n^3 + 3n^3 - 15m^*n^2 + 6m^*n - 2n^2$  produces an infinite sequence of Poulet numbers for any  $m$  non-null positive integer.

### Examples:

Formula becomes  $12n^3 - 17n^2 + 6n$  for  $m = 1$  and we have the following sequence of Poulet numbers  $P = 12n^3 - 17n^2 + 6n$  (obtained for  $n = 5, 11, 23, 29, 35, 65, 71, \dots$ ):

: 1105, 13981, 137149, 278545, 493885, 3224065, 4209661 (...)

Note that all the solutions obtained for  $n$  so far (up to  $n = 71$ ) are of the form  $6k - 1$ .

Formula becomes  $21n^3 - 32n^2 + 12n$  for  $m = 2$  and we have the following sequence of Poulet numbers  $P = 21n^3 - 32n^2 + 12n$  (obtained for  $n = 65, \dots$ ):

: 5632705 (...)

Formula becomes  $30n^3 - 47n^2 + 18n$  for  $m = 3$  and we have the following sequence of Poulet numbers  $P = 30n^3 - 47n^2 + 18n$  (obtained for  $n = 23, 43, 53, 103, \dots$ ):

: 340561, 2299081, 4335241, 32285041 (...)

Note that all the solutions obtained for  $n$  so far (up to  $n = 103$ ) are of the form  $10k + 3$ .

Formula becomes  $39n^3 - 62n^2 + 24n$  for  $m = 4$  and we have the following sequence of Poulet numbers  $P = 39n^3 - 62n^2 + 24n$  (obtained for  $n = 43, \dots$ ):

: 2987167 (...)

Formula becomes  $48n^3 - 77n^2 + 30n$  for  $m = 5$  and we have the following sequence of Poulet numbers  $P = 48n^3 - 77n^2 + 30n$  (obtained for  $n = 29, 37, 77, \dots$ ):

: 1106785, 2327041, 21459361 (...)

Note that all the solutions obtained for  $n$  so far (up to  $n = 77$ ) are of the form  $8k + 5$ .

## 7. A list of thirty-six polynomials and formulas that generate Fermat pseudoprimes

**Abstract.** In this paper I present a simple list of polynomials (in one or two variables) and formulas having the property that they generate Carmichael numbers or Poulet numbers, polynomials and formulas that I have discovered over time.

### Polynomials that generate Carmichael numbers

1.

$$C = (30*n + 7)*(60*n + 13)*(150*n + 31)$$

First six such Carmichael numbers: 2821, 488881, 288120421, 492559141, 776176261, 1632785701 (sequence A182085 in OEIS).

2.

$$C = (30*n - 29)*(60*n - 59)*(90*n - 89)*(180*n - 179)$$

First four such Carmichael numbers: 31146661, 2414829781, 192739365541, 197531244744661 (sequence A182088 in OEIS).

3.

$$C = (330*n + 7)*(660*n + 13)*(990*n + 19)*(1980*n + 37)$$

First two such Carmichael numbers: 63973, 461574735553 (sequence A182089 in OEIS).

4.

$$C = (30*n - 7)*(90*n - 23)*(300*n - 79)$$

First five such Carmichael numbers: 340561, 4335241, 153927961, 542497201, 1678569121 (sequence A182132 in OEIS).

5.

$$C = (30*n - 17)*(90*n - 53)*(150*n - 89)$$

First five such Carmichael numbers: 29341, 1152271, 34901461, 64377991, 775368901 (sequence A182133 in OEIS).

6.

$$C = (60*n + 13)*(180*n + 37)*(300*n + 61)$$

First five such Carmichael numbers: 29341, 34901461, 775368901, 1213619761, 4562359201 (sequence A182416 in OEIS).

### Polynomials that generate Poulet numbers

1.

$$P = 7200*n^2 + 8820*n + 2701$$

First eight such Poulet numbers: 2701, 18721, 49141, 93961, 226801, 314821, 534061, 665281 (sequence A214016 in OEIS).

2.

$$P = 144*n^2 + 222*n + 85$$

First eight such Poulet numbers: 1105, 2047, 3277, 6601, 13747, 16705, 19951, 31417 (sequence A214017 in OEIS).

3.

$$P = 3*(2*n + 1)*(18*n + 11)*(36*n + 17)$$

First four such Poulet numbers: 561, 62745, 656601, 11921001 (sequence A213071 in OEIS).

4.

$$P = (6*m - 1)*((6*m - 2)*n + 1)$$

First eleven such Poulet numbers: 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601 (sequence A210993 in OEIS).

5.

$$P = (6*m + 1)*(6*m*n + 1)$$

First ten such Poulet numbers: 1105, 1387, 1729, 2701, 2821, 4033, 4681, 5461, 6601, 8911 (sequence A214607 in OEIS).

6.

$$P = m*n^2 + (11*m - 23)*n + 19*m - 49$$

First ten such Poulet numbers: 341, 645, 1105, 1387, 2047, 2465, 2821, 3277, 4033, 5461 (sequence A215326 in OEIS).

### Formulas that generate Carmichael numbers

1.

$$C = (30*n - p)*(60*n - (2*p + 1))*(90*n - (3*p + 2)),$$

where  $p$ ,  $2*p + 1$ ,  $3*p + 2$  are all three prime numbers.

First six such Carmichael numbers: 1729, 172081, 294409, 1773289, 4463641, 56052361 (sequence A182087 in OEIS).

Comment: The formula can be reduced to only two possible polynomial forms:  $C = (30*n - 23)*(60*n - 47)*(90*n - 71)$  or  $C = (30*n - 29)*(60*n - 59)*(90*n - 89)$ .

2.

$$C = (p + 30)*(q + 60)*(p*q + 90),$$

where  $p$  and  $q$  are primes.

First two such Carmichael numbers: 488881, 1033669.

### 3.

$C = (30*p + 1)*(60*p + 1)*(90*p + 1)$ ,  
where  $p$  is prime.

First four such Carmichael numbers: 56052361, 216821881, 798770161, 1976295241.

### 4.

$C = p*(2*p - 1)*(3*p - 2)*(6*p - 5)$ ,  
where  $p$  is prime.

First seven such Carmichael numbers: 63973, 31146661, 703995733, 21595159873, 192739365541, 461574735553, 3976486324993 (sequence A182518 in OEIS).

### 5.

$C = p*(2*p - 1)*(n*(2*p - 2) + p)$ ,  
where  $p$  and  $2*p - 1$  are primes.

First ten such Carmichael numbers: 1729, 2821, 41041, 63973, 101101, 126217, 172081, 188461, 294409, 399001 (sequence 182207 in OEIS).

Comment: I conjecture that any Carmichael number  $C$  divisible by  $p$  and  $2*p - 1$  (where  $p$  and  $2*p - 1$  are primes) can be written this way.

### 6.

$C = n*(2*n - 1)*(p*n - p + 1)*(2*p*n - 2*p + 1)$ ,  
where  $p$  is odd and  $n$  natural.

Seven such Carmichael numbers: 63973, 172081, 31146661, 167979421, 277241401, 703995733, 1504651681 (sequence 212882 in OEIS).

### 7.

$C = p*n*(3*p*n + 2)*(6*p*n - 1)$ ,  
where  $p$  is prime and  $n$  natural.

Ten such Carmichael numbers: 2465, 62745, 11119105, 3249390145 (obtained for  $p = 5$ ); 6601 (obtained for  $p = 7$ ); 656601 (obtained for  $p = 11$ ); 41041, 271794601 (obtained for  $p = 13$ ); 11119105, 2159003281 (obtained for  $p = 17$ ) (sequence 212882 in OEIS).

## Formulas that generate Poulet numbers

### 1.

$P = (2^{(3*k + 1)} - 1)/3$ ,  
where  $k$  natural.

First three such Poulet numbers: 341, 1398101, 5726623061.

Comment: The formula can be generalized as  $(n^{(n*k + k + n - 1)} - 1)/(n^2 - 1)$ , formula which generates, I conjecture, an infinity of Fermat pseudoprimes to base  $n$  for any integer  $n$ ,  $n > 1$  (for  $n = 3$  the formula becomes  $(3^{(4*k + 2)} - 1)/8$  and generates Fermat pseudoprimes to base 3 for 14 values of  $k$  from 1 to 20).

2.

$$P = q*((n+1)*q - n*q)*((m+1)*q - m*q),$$

where  $q$  prime and  $m, n$  natural.

Five such Poulet numbers: 10585, 13741, 13981, 29341, 137149.

3.

$$P = q*((n*q - (n+1)*q)*(m*q - (m+1)*q),$$

where  $q$  prime and  $m, n$  natural.

Such Poulet number: 6601.

4.

$$P = q*(q + 2*n)*(q + 2^{2*n} - 2),$$

where  $q$  prime,  $n$  natural:

Two such Poulet numbers: 561, 1105.

5.

$$P = q*(q + 2*n)*(q + 2^{k*n}),$$

where  $q$  prime and  $n, k$  natural.

Four such Poulet numbers: 1729, 2465, 2821, 29341.

6.

$$P = (1 + 2^{k*m})*(1 + 2^{k*n})*(1 + 2^{k*(m+n)}),$$

where  $k, m, n$  natural.

Two such Poulet numbers: 13981, 252601.

7.

$$P = 3*(3 + 2^k)*(3 + q*2^h),$$

where  $q$  prime and  $k, h$  natural.

Three such Poulet numbers: 645, 1905, 8481.

8.

$$P = q^2 + 81*q + 3*q*r,$$

where  $q, r$  primes.

Four such Poulet numbers: 2821, 6601, 14491, 19951.

Comment: Note that the numbers (2821, 6601) and (14491, 19951) are “pairs” because  $2821 = 13^2 + 81*13 + 3*13*41$  while  $6601 = 41^2 + 81*41 + 3*13*41$  and also the values of the  $[q, r]$  for 14491 and 19951 are  $[43, 71]$  respectively  $[71, 43]$ .

9.

$$P = r*q*(n*(q-1) + r),$$

where  $r, q$  primes and  $n$  natural.

Six such Poulet numbers: 137149, 340561, 852841, 950797, 1052503, 1357621.

Comment: I conjecture that any Poulet number having as prime factors both the numbers 23 and 67 can be written this way, also any Poulet number having as prime factors both the numbers 11 and 61.

### 10.

$P = 3*q^3*(3*n + 1) - q^2*(15*n + 2) + 6*q*n$ ,  
where  $q$  prime and  $n$  natural.

Six such Poulet numbers: 4335241, 13421773, 17316001, 17098369, 93869665, 170640961.

Comment: I conjecture that any Poulet number having as prime factors both a number of the form  $30*k + 23$  and a number of the form  $90*k + 67$  can be written this way.

### 11.

$P = 6*q^3*(6*n + 1) - q^2*(66*n + 5) + 30*q*n$ ,  
where  $q$  prime and  $n$  natural.

Six such Poulet numbers: 5148001, 7519441, 9890881, 12262321.

Comment: I conjecture that any Poulet number having as prime factors both a number of the form  $30*k + 11$  and a number of the form  $180*k + 61$  can be written this way.

### 12.

$P = ((2^n)^k)*((2^n)^{(k+1)} + 2^{n+1}) + 1$ ,  
where  $k, n$  natural.

Ten such Poulet numbers: 561, 33153 (obtained for  $n = 1$ ); 1105, 16705 (obtained for  $n = 2$ ); 4369, 1052929, 268505089 (obtained for  $n = 4$ ), 266305 (obtained for  $n = 6$ ); 2113665 (obtained for  $n = 7$ ); 16843009 (obtained for  $n = 8$ ).

### 13.

$P = 2*q^2 - q$ ,  
where  $q$  is also a Poulet number.

First six such Poulet numbers: 831405, 5977153, 15913261, 21474181, 38171953, 126619741 (sequence A215343 in OEIS).

### 14.

$P = (q^2 + 2*q)/3$ ,  
where  $q$  is also a Poulet number.

First six such Poulet numbers: 997633, 1398101, 3581761, 26474581, 37354465, 63002501 (sequence A216276 in OEIS).

### 15.

$P = q^2*n - q*n + p$ ,  
where  $q$  is also a Poulet number and  $n$  natural.

First six such Poulet numbers: 348161, 831405, 1246785, 1275681, 2077545, 2513841 (sequence A217835 in OEIS).

**16.**

$P = (n^m + n^*m)/(m + 1)$ ,  
where  $m, n$  natural.

Ten such Poulet numbers: 341, 645, 2465, 2821, 4033 (obtained for  $m = 2$ ); 341, 1729, 188461, 228241, 1082809 (obtained for  $m = 3$ ) (sequence A216170 in OEIS).

**17.**

$P = (6*k - 1)*((6*k - 2)*n + 1)$ ,  
where  $k, n$  natural.

First eleven such Poulet numbers: 341, 561, 645, 1105, 1905, 2047, 2465, 3277, 4369, 4371, 6601 (sequence A210993 in OEIS).

## 8. A list of 15 sequences of Poulet numbers based on the multiples of the number 6

**Abstract.** In previous papers, I presented few applications of the multiples of the number 30 in the study of Carmichael numbers, i.e. in finding possible infinite sequences of such numbers; in this paper I shall list 15 probably infinite sequences of Poulet numbers that I discovered based on the multiples of the number 6.

### A list with 15 probably infinite sequences of Poulet numbers based on the multiples of the number 6.

(1) Poulet numbers of the form

$$P = (6*n + 7)*(12*n + 13).$$

First 4 terms: 2701 (= 37\*73), 8911 (= 7\*19\*67), 10585 (= 5\*29\*73), 18721 (= 97\*193),  
obtained for  $n = 5, 10, 11$ .

(2) Poulet numbers of the form

$$P = (6*n + 7)*(30*n + 31).$$

First 6 terms: 1729 (= 7\*13\*19), 4681 (= 31\*151), 30889 (= 17\*23\*157), 41041 (= 7\*11\*13\*41), 46657 (= 13\*37\*97), 52633 (= 7\*73\*103),  
obtained for  $n = 2, 4, 12, 16$ .

(3) Poulet numbers of the form

$$P = (12*n + 13)*(30*n + 31).$$

First term: 23377 (= 97\*241),  
obtained for  $n = 7$ .

(4) Poulet numbers of the form

$$P = (6*n + 7)*(12*n + 13)*(30*n + 31).$$

First 5 terms: 2821 (= 7\*13\*31), 63973 (= 7\*13\*19\*37), 285541 (= 31\*61\*151), 488881 (= 37\*73\*181), 7428421 (= 7\*11\*13\*41\*181),  
obtained for  $n = 0, 2, 4, 5, 14$ .

Conjecture: The number  $(6*n + 7)*(12*n + 13)*(30*n + 31)$  is a Poulet number if (but not only if)  $6*n + 7$ ,  $12*n + 13$  and  $30*n + 31$  are all three prime numbers.

(5) Poulet numbers of the form

$$P = (6*n + 1)*(12*n + 1).$$

First 4 terms: 2701 (= 37\*73), 8911 (= 7\*19\*67), 10585 (= 5\*29\*73), 18721 (= 97\*193),  
obtained for  $n = 6, 11, 12, 16$ .

(6) Poulet numbers of the form

$$P = (6*n + 1)*(18*n + 1).$$

First 4 terms: 2821 (= 7\*13\*31), 4033 (= 37\*109), 5461 (43\*127), 15841 (= 7\*31\*73),  
obtained for  $n = 5, 6, 7, 12$ .

(7) Poulet numbers of the form

$$P = (12*n + 1)*(18*n + 1).$$

First term: 7957 (73\*109),  
obtained for  $n = 6$ .

(8) Poulet numbers of the form

$$P = (6*n + 1)*(12*n + 1)*(18*n + 1).$$

First 6 terms: 1729 (= 7\*13\*19), 172081 (= 7\*13\*31\*61), 294409 (= 37\*73\*109), 464185 (= 5\*17\*43\*127), 1773289 (= 67\*133\*199), 4463641 (= 7\*13\*181\*271),  
obtained for n = 1, 5, 6, 7, 11, 15.

Note: The numbers  $(6*n + 1)*(12*n + 1)*(18*n + 1)$ , when  $6*n + 1$ ,  $12*n + 1$  and  $18*n + 1$  are all three primes, are the well known Chernick numbers, so of course they are consequently Poulet numbers, but note that there exist such numbers which are Poulet numbers though  $6*n + 1$ ,  $12*n + 1$  and  $18*n + 1$  are not all three primes.

(9) Poulet numbers of the form

$$P = (6*n + 1)*(12*n + 1)*(18*n + 1)*(36*n + 1).$$

First 4 terms: 63973 (= 7\*13\*19\*37), 31146661 (= 7\*13\*31\*61\*181), 703995733 (= 7\*19\*67\*199\*397), 2414829781 (= 7\*13\*181\*271\*541),  
obtained for n = 1, 5, 11, 15.

Note: The numbers  $(6*n + 1)*(12*n + 1)*(18*n + 1)*(36*n + 1)$ , when  $6*n + 1$ ,  $12*n + 1$ ,  $18*n + 1$  and  $36*n + 1$  are all four primes, are known that are Carmichael numbers, so of course they are consequently Poulet numbers, but note that there exist such numbers which are Poulet numbers though  $6*n + 1$ ,  $12*n + 1$ ,  $18*n + 1$  and  $36*n + 1$  are not all four primes.

(10) Poulet numbers of the form

$$P = (6*n + 1)*(24*n + 1).$$

First 5 terms: 1387 (= 19\*73), 83665 (= 5\*29\*577), 90751 (= 151\*601), 390937 (= 313\*1249), 748657 (= 7\*13\*19\*433),  
obtained for n = 3, 24, 25, 52, 72.

(11) Poulet numbers of the form

$$P = (6*n - 1)*(12*n - 3).$$

First 2 terms: 561 (= 3\*11\*17), 4371 (= 3\*31\*47),  
obtained for n = 3, 8.

(12) Poulet numbers of the form

$$P = (6*n - 1)*(18*n - 5).$$

First 3 terms: 341 (= 11\*31), 2465 (5\*17\*29), 8321 (53\*157),  
obtained for n = 2, 5, 9.

(13) Poulet numbers of the form

$$P = (6*n - 1)*(24*n - 7).$$

First 5 terms: 1105 (= 5\*13\*17), 2047 (= 23\*89), 3277 (= 29\*113), 6601 (= 7\*23\*41), 13747 (= 59\*233),  
obtained for n = 3, 4, 5, 7, 10.

(14) Poulet numbers of the form

$$P = (6*n - 1)*(18*n - 5)*(60*n - 19).$$

First 2 terms: 340561 (= 13\*17\*23\*67), 4335241 (= 53\*157\*521),  
obtained for n = 4, 9.

(15) Poulet numbers of the form

$$P = (6*n + 1)*(18*n + 1)*(30*n + 1).$$

First 2 terms: 29341 (= 13\*37\*61), 1152271 (= 43\*127\*211), obtained for n = 2, 7.

## 9. Bold conjecture on Fermat pseudoprimes

**Abstract.** In many of my previous papers I showed various methods, formulas and polynomials designed to generate sequences, possible infinite, of Poulet numbers or Carmichael numbers. In this paper I state that there exist a method to place almost any Fermat pseudoprime to base two (Poulet number) in such a sequence, as a further term or as a starting term.

### Conjecture:

If the prime factors of a Poulet number not divisible with 3 can be expressed in the following way, i.e. the least from them  $P$  as  $6^n + 1$ ,  $6^n - 1$ ,  $6^n + 5$  or  $6^n - 5$  and the product of the rest of them  $Q$  as  $6^m + 1$ ,  $6^m - 1$ ,  $6^m + 5$  or  $6^m - 5$ , then there exist an infinity of Poulet numbers of the form  $P*Q$ .

As example, the first Poulet number, 341, is equal to  $11*31$  and we have  $11 = 6^2 - 1$  (so  $6^n - 1$ ) and  $31 = 18*2 - 5$  (so  $18^n - 5$ ); the conjecture states that there exist an infinity of Poulet numbers of the form  $(6^n - 1)*(18^n - 5)$ .

Note that not any Poulet number not divisible by 3 (though the most of them) can be written the way described above; as example, the 2-Poulet number  $3277 = 29*113$  (29 is equal to  $6^4 + 5$  and to  $6^5 - 1$ , but 113 isn't equal either to  $6^4 + 1$  or  $6^5 - 1$  neither with  $6^4 + 5$  or  $6^5 - 5$ ).

### Examples:

(for few from the first Poulet numbers not divisible by 3)

- :  $341 = 11*31$  is the starting term, obtained for  $n = 2$ , in the sequence of Poulet numbers  $(6^n - 1)*(18^n - 5)$  which has the following terms: 2465, 8321, 83333, 162401 (...) obtained for  $n = 5, 9, 28, 39$  (...);
- :  $1105 = 5*13*17 = 5*221$  is the starting term, obtained for  $n = 1$ , in the sequence of Poulet numbers  $(6^n - 1)*(22*2^n - 1)$  which has the following terms: 11305 (...) obtained for  $n = 3$  (...);
- :  $1387 = 19*73$  is the starting term, obtained for  $n = 3$ , in the sequence of Poulet numbers  $(6^n + 1)*(24*2^n + 1)$  which has the following terms: 83665, 90751 (...) obtained for  $n = 24, 25$  (...);
- :  $1729 = 7*13*19 = 7*247$  is the starting term, obtained for  $n = 1$ , in the sequence of Poulet numbers  $(6^n + 1)*(246*2^n + 1)$  which has the following terms: 1082809, 1615681 (...) obtained for  $n = 27, 33$  (...);
- :  $2047 = 23*89$  is the starting term, obtained for  $n = 3$ , in the sequence of Poulet numbers  $(6^n + 5)*(30*2^n - 1)$ ;

- :  $2465 = 5 \cdot 17 \cdot 19 = 5 \cdot 493$  is the starting term, obtained for  $n = 1$ , in the sequence of Poulet numbers  $(6 \cdot n - 1) \cdot (492 \cdot n + 1)$ ;
- :  $2701 = 37 \cdot 73$  is the starting term, obtained for  $n = 6$ , in the sequence of Poulet numbers  $(6 \cdot n + 1) \cdot (12 \cdot n + 1)$  which has the following terms: 8911, 10585, 18721 (...) obtained for  $n = 11, 12, 16$  (...);

- :  $2821 = 7 \cdot 13 \cdot 31 = 7 \cdot 403$  is the starting term, obtained for  $n = 1$ , in the sequence of Poulet numbers  $(6 \cdot n + 1) \cdot (402 \cdot n + 1)$ ;

- :  $4033 = 37 \cdot 109$  is the second term, obtained for  $n = 6$ , in the sequence of Poulet numbers  $(6 \cdot n + 1) \cdot (18 \cdot n + 1)$  which has as the first term, obtained for  $n = 5$ , the Poulet number 2821, and as the following terms: 5461, 15841 (...) obtained for  $n = 7, 12$  (...);

Note that the Poulet number 2821 is a term in both of the distinct sequences  $(6 \cdot n + 1) \cdot (402 \cdot n + 1)$  and  $(6 \cdot n + 1) \cdot (18 \cdot n + 1)$ .

- :  $4369 = 17 \cdot 257$  is the starting term, obtained for  $n = 2$ , in the sequence of Poulet numbers  $(6 \cdot n + 5) \cdot (126 \cdot n + 5)$ ;
- :  $4681 = 31 \cdot 151$  is the second term, obtained for  $n = 5$ , in the sequence of Poulet numbers  $(6 \cdot n + 1) \cdot (30 \cdot n + 1)$  which has as the first term, obtained for  $n = 3$ , the Poulet number 1729, and as the following terms: 41041, 46657, 52633 (...) obtained for  $n = 15, 16, 17$  (...);

Note that the Poulet number 1729 is a term in both of the distinct sequences  $(6 \cdot n + 1) \cdot (246 \cdot n + 1)$  and  $(6 \cdot n + 1) \cdot (30 \cdot n + 1)$ .

- :  $5461 = 43 \cdot 127$  is the second term, obtained for  $n = 7$ , in the sequence of Poulet numbers  $(6 \cdot n + 1) \cdot (18 \cdot n + 1)$  which has as the first term, obtained for  $n = 5$ , the Poulet number 2821.

## 10. Another bold conjecture on Fermat pseudoprimes

**Abstract.** In my previous paper “Bold conjecture on Fermat pseudoprimes” I stated that there exist a method to place almost any Fermat pseudoprime to base two (Poulet number) in an infinite subsequence of such numbers, defined by a quadratic polynomial, as a further term or as a starting term of such a sequence. In this paper I conjecture that there is yet another way to place a Poulet number in such a sequence defined by a polynomial, this time not necessarily quadratic.

### Conjecture:

If we express the prime factors of a Poulet number, not divisible by 3,  $P = d_1 * d_2 * \dots * d_i$ , where  $d_1, d_2, \dots, d_i$  are its prime factors, as  $P = (2n + 1) * (m_1 * n + 1) * \dots * (m_i * n + 1)$ , then there exist an infinity of Poulet numbers of this form.

As example, the first Poulet number, 341, is equal to  $11 * 31$  and we have  $11 = 2 * 5 + 1$  (so  $2 * n + 1$ ) and  $31 = 6 * 5 + 1$  (so  $6 * n + 1$ ); the conjecture states that there exist an infinity of Poulet numbers of the form  $(2 * n + 1) * (6 * n + 1)$ .

Note that not any Poulet number not divisible by 3 (though the most of them) can be written the way described above; as example, the 2-Poulet number  $6601 = 7 * 23 * 41$  (7 is equal to  $2 * 3 + 1$ , but 23 isn't equal to  $m * 3 + 1$  neither 41).

### Examples:

(for few from the first Poulet numbers not divisible by 3)

- :  $341 = 11 * 31$  is the starting term, obtained for  $n = 5$ , in the sequence of Poulet numbers  $(2 * n + 1) * (6 * n + 1)$  which has the following terms: 645, 2465, 2821, 4033 (...) obtained for  $n = 7, 14, 15, 18$  (...);
- :  $1105 = 5 * 13 * 17$  is the starting term, obtained for  $n = 2$ , in the sequence of Poulet numbers  $(2 * n + 1) * (6 * n + 1) * (8 * n + 1)$  which has the following terms: 13981, 137149, 278545, 493885 (...) obtained for  $n = 5, 11, 14, 17$  (...);
- :  $1387 = 19 * 73$  is the second term, obtained for  $n = 9$ , in the sequence of Poulet numbers  $(2 * n + 1) * (8 * n + 1)$  which has as the first term, obtained for  $n = 8$ , the Poulet number 1105, and as the following terms: 2047, 3277, 6601 (...) obtained for  $n = 11, 14, 20$  (...);
- :  $1729 = 7 * 13 * 19$  is the starting term, obtained for  $n = 3$ , in the sequence of Poulet numbers  $(2 * n + 1) * (4 * n + 1) * (6 * n + 1)$  which has the following terms: 18705, 172081, 294409 (...) obtained for  $n = 7, 15, 18$  (...);
- :  $2047 = 23 * 89$  is the third term, obtained for  $n = 11$ , in the sequence of Poulet numbers  $(2 * n + 1) * (8 * n + 1)$  which has as the first and second terms, obtained for  $n = 8$  and  $n = 9$ , the Poulet numbers 1105 and 1387;

- :  $2465 = 5 \cdot 17 \cdot 29$  is the starting term, obtained for  $n = 2$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (8 \cdot n + 1) \cdot (14 \cdot n + 1)$  which has the following terms: 176149 (...) obtained for  $n = 9$  (...);
- :  $2701 = 37 \cdot 73$  is the second term, obtained for  $n = 18$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (4 \cdot n + 1)$  which has as the first term, obtained for  $n = 8$ , the Poulet number 561;
- :  $2821 = 7 \cdot 13 \cdot 31$  is the starting term, obtained for  $n = 3$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (4 \cdot n + 1) \cdot (10 \cdot n + 1)$  which has the following terms: 63973, 285541, 488881 (...) obtained for  $n = 9, 15, 18$  (...);
- :  $4033 = 37 \cdot 109$  is the fifth term, obtained for  $n = 18$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (6 \cdot n + 1)$  which has as the previous terms, obtained for  $n = 5, 7, 14, 15$  (...) the Poulet numbers 341, 645, 2465, 2821 (...);
- :  $4369 = 17 \cdot 257$  is the starting term, obtained for  $n = 8$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (32 \cdot n + 1)$ ;
- :  $4681 = 31 \cdot 151$  is the third term, obtained for  $n = 15$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (10 \cdot n + 1)$  which has as the first and second terms, obtained for  $n = 5$  and  $n = 9$ , the Poulet numbers 561 and 1729;
- :  $5461 = 43 \cdot 127$  is a term, obtained for  $n = 21$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (6 \cdot n + 1)$ ;
- :  $7957 = 73 \cdot 109$  is a term, obtained for  $n = 36$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (3 \cdot n + 1)$ ;
- :  $8321 = 53 \cdot 157$  is a term, obtained for  $n = 26$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (6 \cdot n + 1)$ ;
- :  $8911 = 7 \cdot 19 \cdot 67$  is the first term, obtained for  $n = 3$ , in the sequence of Poulet numbers  $(2 \cdot n + 1) \cdot (6 \cdot n + 1) \cdot (22 \cdot n + 1)$  which has the following terms: 63973 (...) obtained for  $n = 6$  (...).

## 11. Generic form of the Poulet numbers having a prime factor of the form $30n + 23$

**Abstract.** In this paper I observe that many Poulet numbers  $P$  having a prime factor  $q$  of the form  $30n + 23$ , where  $n$  positive integer, can be written as  $P = m(q^2 - q) + q^2$ , where  $m$  positive integer, and I conjecture that any Poulet number  $P$  having 23 as a prime factor can be written as  $P = 506m + 529$ , where  $m$  positive integer.

### Observation:

Many Poulet numbers  $P$  having a prime factor  $q$  of the form  $30n + 23$ , where  $n$  positive integer, can be written as  $P = m(q^2 - q) + q^2$ , where  $m$  positive integer.

Examples:

- :  $8321 = 53 \cdot 157$  and  $8321 = 2 \cdot (53^2 - 53) + 53^2$ , so  $[q, m] = [53, 2]$ ;
- :  $85489 = 53 \cdot 1613$  and  $85489 = 30 \cdot (53^2 - 53) + 53^2$ , so  $[q, m] = [53, 30]$ ;
- :  $88561 = 11 \cdot 83 \cdot 97$  and  $88561 = 12 \cdot (83^2 - 83) + 83^2$ , so  $[q, m] = [83, 12]$ ;
- :  $91001 = 17 \cdot 53 \cdot 101$  and  $91001 = 32 \cdot (53^2 - 53) + 53^2$ , so  $[q, m] = [53, 32]$ ;
- :  $208465 = 5 \cdot 173 \cdot 241$  and  $208465 = 6 \cdot (173^2 - 173) + 173^2$ , so  $[q, m] = [173, 6]$ ;
- :  $215265 = 3 \cdot 5 \cdot 113 \cdot 127$  and  $215265 = 16 \cdot (113^2 - 113) + 113^2$ , so  $[q, m] = [113, 16]$ ;
- :  $275887 = 263 \cdot 1049$  and  $275887 = 3 \cdot (263^2 - 263) + 263^2$ , so  $[q, m] = [263, 3]$ ;
- :  $278545 = 5 \cdot 17 \cdot 29 \cdot 113$  and  $278545 = 21 \cdot (113^2 - 113) + 113^2$ , so  $[q, m] = [113, 21]$ ;
- :  $422659 = 3 \cdot 53 \cdot 2687$  and  $422659 = 154 \cdot (53^2 - 53) + 53^2$ , so  $[q, m] = [53, 154]$ .

### Conjecture:

Any Poulet number  $P$  having 23 as a prime factor can be written as  $P = 506m + 529$ , where  $m$  positive integer.

### Verifying the conjecture:

(For the first seven such Poulet numbers)

- :  $2047 = 23 \cdot 89$  and  $2047 = 3 \cdot 506 + 529$ , so  $m = 3$ ;
- :  $6601 = 7 \cdot 23 \cdot 41$  and  $6601 = 12 \cdot 506 + 529$ , so  $m = 12$ ;
- :  $15709 = 23 \cdot 683$  and  $15709 = 30 \cdot 506 + 529$ , so  $m = 30$ ;
- :  $30889 = 17 \cdot 23 \cdot 79$  and  $30889 = 60 \cdot 506 + 529$ , so  $m = 60$ ;
- :  $137149 = 23 \cdot 67 \cdot 89$  and  $137149 = 270 \cdot 506 + 529$ , so  $m = 270$ ;
- :  $272251 = 7 \cdot 19 \cdot 23 \cdot 89$  and  $272251 = 537 \cdot 506 + 529$ , so  $m = 537$ ;
- :  $340561 = 13 \cdot 17 \cdot 23 \cdot 67$  and  $340561 = 672 \cdot 506 + 529$ , so  $m = 672$ .

Note the following 13 Poulet numbers having a prime factor of the form  $23n + 30$  (from the first 29 such Poulet numbers) which can't be written in the way showed above:  $13747 = 59 \cdot 233$ ,  $3277 = 29 \cdot 113$ ,  $31417 = 89 \cdot 353$ ,  $60787 = 89 \cdot 683$ ,  $65077 = 59 \cdot 1103$ ,  $72885 = 3 \cdot 5 \cdot 43 \cdot 113$ ,  $88357 = 149 \cdot 593$ ,  $130561 = 137 \cdot 953$ ,  $194221 = 167 \cdot 1163$ ,  $196021 = 7 \cdot 41 \cdot 683$ ,  $253241 = 157 \cdot 1613$ ,  $256999 = 233 \cdot 1103$ ,  $280601 = 277 \cdot 1013$ . In all of these cases, the prime factor of the form  $23n + 30$  is the biggest prime factor (a particular case is the number 256999 having both of the factors of the form  $23n + 30$ ).

## 12. Notable observation on a property of Carmichael numbers

**Abstract.** In this paper I conjecture that for any Carmichael number  $C$  is true one of the following two statements: (i) there exist at least one prime  $q$ ,  $q$  lesser than  $\text{Sqr}(C)$ , such that  $p = (C - q)/(q - 1)$  is prime, power of prime or semiprime  $m \cdot n$ ,  $n > m$ , with the property that  $n - m + 1$  is prime or power of prime or  $n + m - 1$  is prime or power of prime; (ii) there exist at least one prime  $q$ ,  $q$  lesser than  $\text{Sqr}(C)$ , such that  $p = (C - q)/((q - 1) \cdot 2^n)$  is prime or power of prime. In two previous papers I made similar assumptions on the squares of primes of the form  $10k + 1$  respectively  $10k + 9$  and I always believed that Fermat pseudoprimes behave in several times like squares of primes.

### Conjecture:

For any Carmichael number  $C$  is true one of the following two statements:

- (i) there exist at least one prime  $q$ ,  $q$  lesser than  $\text{Sqr}(C)$ , such that  $p = (C - q)/(q - 1)$  is prime, power of prime or semiprime  $m \cdot n$ ,  $n > m$ , with the property that  $n - m + 1$  is prime or power of prime or  $n + m - 1$  is prime or power of prime;
- (ii) there exist at least one prime  $q$ ,  $q$  lesser than  $\text{Sqr}(C)$ , such that  $p = (C - q)/((q - 1) \cdot 2^n)$  is prime or power of prime.

### Verifying the conjecture:

(for the first ten Carmichael numbers)

- :  $C = 561$  and  $(C - 5)/4 = 139$ , prime; also  $(C - 11)/10 = 5 \cdot 11$ , semiprime such that  $11 - 5 + 1 = 7$ , prime; also  $(C - 17)/(16 \cdot 2) = 17$ , prime;
- :  $C = 1105$  and  $(C - 7)/6 = 3 \cdot 61$ , semiprime such that  $61 - 3 + 1 = 59$ , prime; also  $(C - 13)/12 = 7 \cdot 13$ , semiprime such that  $13 - 7 + 1 = 7$ , prime and  $13 + 7 - 1 = 19$ , prime; also  $(C - 17)/(16 \cdot 2^2) = 17$ , prime;
- :  $C = 1729$  and  $(C - 5)/4 = 431$ , prime; also  $(C - 17)/16 = 107$ , prime; also  $(C - 37)/36 = 47$ , prime;
- :  $C = 2465$  and  $(C - 29)/28 = 3 \cdot 29$ , semiprime such that  $29 - 3 + 1 = 27 = 3^3$ , power of prime and  $29 + 3 - 1 = 31$ , prime;
- :  $C = 2821$  and  $(C - 7)/6 = 7 \cdot 67$ , semiprime such that  $67 - 7 + 1 = 61$ , prime and  $67 + 7 - 1 = 73$ , prime; also  $(C - 11)/10 = 281$ , prime; also  $(C - 31)/30 = 3 \cdot 31$ , semiprime such that  $31 - 3 + 1 = 29$ , prime;
- :  $C = 6601$  and  $(C - 5)/4 = 17 \cdot 97$ , semiprime such that  $97 - 17 + 1 = 81 = 3^4$ , power of prime and  $97 + 17 - 1 = 113$ , prime; also  $(C - 7)/6 = 7 \cdot 157$ , semiprime such that  $157 - 7 + 1 = 151$ , prime and  $157 + 7 - 1 = 163$ , prime; also  $(C - 11)/10 = 659$ , prime; also  $(C - 23)/22 = 13 \cdot 23$ , semiprime such that  $23 - 13 + 1 = 11$ , prime; also  $(C - 31)/30 = 3 \cdot 73$ , semiprime such that  $73 - 3 + 1 = 71$ , prime; also  $(C - 61)/60 = 109$ , prime;
- :  $C = 8911$  and  $(C - 23)/(22 \cdot 2^2) = 101$ , prime; also  $(C - 31)/(30 \cdot 2^3) = 37$ , prime; also  $(C - 67)/(66 \cdot 2) = 67$ , prime;

- :  $C = 10585$  and  $(C - 7)/6 = 41*43$ , semiprime such that  $43 - 41 + 1 = 3$ , prime and  $43 + 1 - 1 = 83$ , prime; also  $(C - 13)/12 = 881$ , prime; also  $(C - 19)/18 = 587$ , prime; also  $(C - 29)/28 = 13*29$ , semiprime such that  $29 - 13 + 1 = 17$ , prime and  $13 + 29 - 1 = 41$ , prime; also  $(C - 37)/36 = 293$ , prime; also  $(C - 43)/42 = 251$ , prime; also  $(C - 73)/(73*2) = 73$ , prime;
- :  $C = 5841$  and  $(C - 5)/4 = 37*107$ , semiprime such that  $107 - 37 + 1 = 71$ , prime; also  $(C - 11)/10 = 1583$ , prime; also  $(C - 13)/12 = 1319$ , prime; also  $(C - 61)/60 = 263$ , prime; also  $(C - 67)/66 = 239$ , prime; also  $(C - 73)/72 = 3*73$ , semiprime such that  $73 - 3 + 1 = 71$ , prime; also  $(C - 89)/88 = 179$ , prime; also  $(C - 97)/(96*2^2) = 41$ , prime;
- :  $C = 29341$  and  $(C - 7)/6 = 4889$ , prime; also  $(C - 31)/30 = 977$ , prime; also  $(C - 61)/(60*2^3) = 61$ , prime.

### 13. Conjecture which states that any Carmichael number can be written in a certain way

**Abstract.** In this paper we conjecture that any Carmichael number  $C$  can be written as  $C = (p + 270) \cdot (n + 1) - n$ , where  $n$  non-null positive integer and  $p$  prime. Incidentally, verifying this conjecture, we found results that encouraged us to issue yet another conjecture, i.e. that there exist an infinity of Poulet numbers  $P_2$  that could be written as  $(P_1 + n)/(n + 1) - 270$ , where  $n$  is non-null positive integer and  $P_1$  is also a Poulet number.

#### Conjecture:

In this paper we conjecture that any Carmichael number  $C$  can be written as  $C = (p + 270) \cdot (n + 1) - n$ , where  $n$  non-null positive integer and  $p$  prime.

#### Verifying the conjecture:

(for the first eight Carmichael numbers)

- :  $561 = (11 + 270) \cdot 2 - 1$ , so  $[n, p] = [1, 11]$ ;
- :  $1105 = (283 + 270) \cdot 2 - 1$ , so  $[n, p] = [1, 283]$ ; also  $1105 = (7 + 270) \cdot 4 - 3$ , so  $[n, p] = [3, 7]$ ;
- :  $1729 = (307 + 270) \cdot 3 - 2$ , so  $[n, p] = [2, 307]$ ; also  $1729 = (163 + 270) \cdot 4 - 3$ , so  $[n, p] = [3, 163]$ ; also  $1729 = (19 + 270) \cdot 6 - 5$ , so  $[n, p] = [5, 19]$ ;
- :  $2465 = (347 + 270) \cdot 4 - 3$ , so  $[n, p] = [3, 347]$ ; also  $2465 = (83 + 270) \cdot 7 - 6$ , so  $[n, p] = [6, 83]$ ;
- :  $2821 = (941 + 270) \cdot 3 - 2$ , so  $[n, p] = [2, 941]$ ; also  $2821 = (13 + 270) \cdot 10 - 9$ , so  $[n, p] = [9, 13]$ ;
- :  $6601 = (1931 + 270) \cdot 3 - 2$ , so  $[n, p] = [2, 1931]$ ; also  $6601 = (1381 + 270) \cdot 4 - 3$ , so  $[n, p] = [3, 1381]$ ; also  $6601 = (1051 + 270) \cdot 5 - 4$ , so  $[n, p] = [4, 1051]$ ; also  $6601 = (331 + 270) \cdot 11 - 10$ , so  $[n, p] = [10, 331]$ ; also  $6601 = (281 + 270) \cdot 12 - 11$ , so  $[n, p] = [11, 281]$ ;
- :  $8911 = (541 + 270) \cdot 11 - 10$ , so  $[n, p] = [10, 541]$ ; also  $8911 = (61 + 270) \cdot 27 - 26$ , so  $[n, p] = [26, 61]$ ;
- :  $10585 = (5023 + 270) \cdot 2 - 1$ , so  $[n, p] = [1, 5023]$ ; also  $10585 = (3529 + 270) \cdot 3 - 2$ , so  $[n, p] = [2, 3529]$ ; also  $10585 = (2377 + 270) \cdot 4 - 3$ , so  $[n, p] = [3, 2377]$ ; also  $10585 = (907 + 270) \cdot 9 - 8$ , so  $[n, p] = [8, 907]$ ; also  $10585 = (613 + 270) \cdot 12 - 11$ , so  $[n, p] = [11, 613]$ ; also  $10585 = (487 + 270) \cdot 14 - 13$ , so  $[n, p] = [13, 487]$ ; also  $10585 = (109 + 270) \cdot 28 - 27$ , so  $[n, p] = [27, 109]$ .

**Note:**

We have not verified, but it would be interesting if the number 1729 would be the first number that could be written as  $C = (p + 270) \cdot (n + 1) - n$ , where  $n$  non-null positive integer and  $p$  prime, in three distinct ways, or if the number 6601 would be the first number that could be written such this in five distinct ways, or if the number 10585 would be the first number that could be written such this in seven distinct ways, or if the first number that could be written such this in  $k$  different ways would be a Carmichael number.

**Conjecture:**

There exist an infinity of Poulet numbers  $P_2$  that could be written as  $(P_1 + n)/(n + 1) - 270$ , where  $n$  is non-null positive integer and  $P_1$  is also a Poulet number.

Example:

:  $701 = (8911 + 2)/3 - 270$ , so  $[n, P_1, P_2] = [2, 8911, 2701]$ .

## 14. Sequence of Poulet numbers obtained by formula $mn - n + 1$ where $m$ of the form $270k + 13$

**Abstract.** In this paper we conjecture that there exist an infinity of Poulet numbers of the form  $m \cdot n - n + 1$ , where  $m$  is of the form  $270 \cdot k + 13$ . Incidentally, verifying this conjecture, we found results that encouraged us to issue yet another conjecture, i.e. that there exist an infinity of numbers  $s$  of the form  $270 \cdot k + 13$  which are semiprimes  $s = p \cdot q$  having the property that  $q - p + 1$  is prime or power of prime.

### Conjecture:

There exist an infinity of Poulet numbers of the form  $m \cdot n - n + 1$ , where  $m$  is of the form  $270 \cdot k + 13$ .

Examples:

: for  $k = 1$ ,  $m = 283$  and the following numbers are Poulet numbers:  
:  $2821 = 283 \cdot 10 - 10 + 1$  (...)  
: for  $k = 2$ ,  $m = 553$  and the following numbers are Poulet numbers:  
:  $1105 = 553 \cdot 2 - 2 + 1$  (...)  
: for  $k = 4$ ,  $m = 1093$  and the following numbers are Poulet numbers:  
:  $3277 = 1093 \cdot 3 - 3 + 1$ ;  
:  $4369 = 1093 \cdot 4 - 4 + 1$ ;  
:  $5461 = 1093 \cdot 5 - 4 + 1$  (...)

The sequence of Poulet numbers of the form  $m \cdot n - n + 1$ , where  $m$  is of the form  $270 \cdot k + 13$ :

: 1105, 2821, 3277, 4369, 5461 (...)

### Conjecture:

There exist an infinity of numbers  $s$  of the form  $270 \cdot k + 13$  which are semiprimes  $s = p \cdot q$  having the property that  $q - p + 1$  is prime or power of prime.

Examples:

: for  $k = 2$ ,  $s = 553 = 7 \cdot 79$  and  $79 - 7 + 1 = 73$ , prime;  
: for  $k = 5$ ,  $s = 1363 = 29 \cdot 47$  and  $47 - 29 + 1 = 19$ , prime;  
: for  $k = 6$ ,  $s = 1633 = 23 \cdot 71$  and  $71 - 23 + 1 = 49$ , power of prime ( $7^2$ );  
: for  $k = 7$ ,  $s = 1903 = 11 \cdot 173$  and  $173 - 11 + 1 = 163$ , prime;  
: for  $k = 8$ ,  $s = 2173 = 41 \cdot 53$  and  $53 - 41 + 1 = 13$ , prime;  
: for  $k = 9$ ,  $s = 2443 = 7 \cdot 349$  and  $349 - 7 + 1 = 343$ , power of prime ( $7^3$ );  
: for  $k = 11$ ,  $s = 2983 = 19 \cdot 157$  and  $157 - 19 + 1 = 139$ , prime;  
: for  $k = 15$ ,  $s = 4063 = 17 \cdot 239$  and  $239 - 17 + 1 = 223$ , prime;  
: for  $k = 16$ ,  $s = 4333 = 7 \cdot 619$  and  $619 - 7 + 1 = 613$ , prime;  
: for  $k = 18$ ,  $s = 4873 = 11 \cdot 443$  and  $443 - 11 + 1 = 433$ , prime;  
: for  $k = 19$ ,  $s = 5143 = 37 \cdot 139$  and  $139 - 37 + 1 = 103$ , prime;  
: for  $k = 24$ ,  $s = 6493 = 43 \cdot 151$  and  $151 - 43 + 1 = 109$ , prime;

: for  $k = 26$ ,  $s = 7033 = 13 \cdot 541$  and  $541 - 13 + 1 = 529$ , power of prime ( $23^2$ );  
 : for  $k = 27$ ,  $s = 7303 = 67 \cdot 109$  and  $109 - 67 + 1 = 43$ , prime;  
 : for  $k = 33$ ,  $s = 8383 = 83 \cdot 101$  and  $101 - 83 + 1 = 19$ , prime;  
 (...)

: for  $k = 20000$ ,  $s = 5400013 = 1627 \cdot 3319$  and  $3319 - 1627 + 1 = 1693$ , prime;  
 (...)

: for  $k = 190000$ ,  $s = 51300013 = 1487 \cdot 34499$  and  $34499 - 1487 + 1 = 33013$ ,  
 prime.

**Note:**

Many other numbers  $s$  of the form  $270 \cdot k + 13$  are semiprimes  $s = p_1 \cdot q_1$  having the property that  $q_1 - p_1 + 1$  is a semiprime  $p_2 \cdot q_2$  having the property that  $q_2 - p_2$  is prime.

Example:

: for  $k = 2000000$ ,  $s = 540000013 = 7 \cdot 77142859$  and  $77142859 - 7 + 1 = 77142853 = 41 \cdot 1881533$  and  $1881533 - 41 + 1 = 1881493$ , prime.

## 15. Two conjectures on Super-Poulet numbers with two respectively three prime factors

**Abstract.** In this paper I make two conjectures on Super-Poulet numbers with two, respectively three prime factors.

### Definition:

Super-Poulet numbers are the Poulet numbers whose divisors  $d$  all satisfy the relation  $d$  divides  $2^d - 2$  (see the sequence A050217 in OEIS for the list of Super-Poulet numbers).

### Note:

Every 2-Poulet number (Poulet number with only two prime factors) is also a Super-Poulet number (see the sequence A214305 for the list of 2-Poulet numbers).

### Conjecture 1:

For any 2-Poulet number  $q \cdot r$  (obviously  $q$  and  $r$  primes, distinct ( $q < r$ ) beside the case of the two 2-Poulet numbers which are the squares of the two known Wieferich primes) is true one of the following two statements:

- i) there exist  $n$  positive integer such that  $r = n \cdot q - n + 1$ ;
- ii) there exist  $p$  prime,  $p$  greater than 7, also  $n$  and  $m$  positive integers, such that  $q = n \cdot p - n + 1$  and  $r = m \cdot p - m + 1$ .

### Verifying the conjecture:

(For the first twenty-two 2-Poulet numbers)

:  $341 = 11 \cdot 31$  and  $31 = 11 \cdot 3 - 2$ ;  
:  $1387 = 19 \cdot 73$  and  $73 = 19 \cdot 4 - 3$ ;  
:  $2047 = 23 \cdot 89$  and  $89 = 23 \cdot 4 - 3$ ;  
:  $2701 = 37 \cdot 73$  and  $73 = 37 \cdot 2 - 1$ ;  
:  $3277 = 29 \cdot 113$  and  $113 = 29 \cdot 4 - 3$ ;  
:  $4033 = 37 \cdot 109$  and  $109 = 37 \cdot 3 - 2$ ;  
:  $4369 = 17 \cdot 257$  and  $257 = 17 \cdot 6 - 5$ ;  
:  $4681 = 31 \cdot 151$  and  $151 = 31 \cdot 5 - 4$ ;  
:  $5461 = 43 \cdot 127$  and  $127 = 43 \cdot 3 - 2$ ;  
:  $7957 = 73 \cdot 109$  and  $73 = 6 \cdot 13 - 5$  while  $109 = 9 \cdot 13 - 8$ ;  
:  $8321 = 53 \cdot 157$  and  $157 = 53 \cdot 3 - 2$ ;  
:  $10261 = 31 \cdot 331$  and  $331 = 31 \cdot 11 - 10$ ;  
:  $13747 = 59 \cdot 233$  and  $233 = 59 \cdot 4 - 3$ ;  
:  $14491 = 43 \cdot 337$  and  $337 = 43 \cdot 8 - 7$ ;  
:  $15709 = 23 \cdot 683$  and  $683 = 23 \cdot 31 - 30$ ;  
:  $18721 = 97 \cdot 193$  and  $193 = 97 \cdot 2 - 1$ ;  
:  $19951 = 71 \cdot 281$  and  $281 = 71 \cdot 4 - 3$ ;  
:  $23377 = 97 \cdot 241$  and  $97 = 6 \cdot 17 - 5$  while  $241 = 15 \cdot 17 - 14$ ;  
:  $31417 = 89 \cdot 353$  and  $353 = 89 \cdot 4 - 3$ ;

- :  $31609 = 73 \cdot 433$  and  $433 = 73 \cdot 6 - 5$ ;
- :  $31621 = 103 \cdot 307$  and  $307 = 103 \cdot 3 - 2$ ;
- :  $35333 = 89 \cdot 397$  and  $89 = 4 \cdot 23 - 3$  while  $397 = 18 \cdot 23 - 17$ .

Note that the conjecture is obviously true for the case of the two 2-Poulet numbers which are the squares of the two known Wieferich primes, i.e.  $1194649 = 1093^2$  and  $12327121 = 3511^2$ . For instance, the prime 1093 can be written in seven distinct ways like  $n \cdot p - p + 1$ , where  $p$  prime:  $1093 = 2 \cdot 547 - 1 = 7 \cdot 157 - 6 = 14 \cdot 79 - 13 = 21 \cdot 53 - 20 = 26 \cdot 43 - 25 = 39 \cdot 29 - 38 = 197 \cdot 7 - 6$  (and, of course,  $1093 = 1093 \cdot 1 - 0$ ).

## Conjecture 2:

For any Super-Poulet number with three prime factors  $p \cdot q \cdot r$  (obviously  $p, q$  and  $r$  primes,  $p < q < r$ ) is true one of the following two statements:

- iii) there exist  $n$  and  $m$  positive integers such that  $q = n \cdot p - n + 1$  and  $r = m \cdot p - m + 1$ ;
- iv) there exist  $s$  prime,  $s$  greater than 7, also  $a, b$  and  $c$  positive integers, such that  $p = a \cdot s - a + 1$ ,  $q = b \cdot s - b + 1$  and  $r = c \cdot s - c + 1$ .

## Verifying the conjecture:

(For the first 18 such Super-Poulet numbers)

- :  $294409 = 37 \cdot 73 \cdot 109$  and  $73 = 37 \cdot 2 - 1$  while  $109 = 37 \cdot 3 - 2$ ;
- :  $1398101 = 23 \cdot 89 \cdot 683$  and  $89 = 23 \cdot 4 - 3$  while  $683 = 23 \cdot 31 - 30$ ;
- :  $1549411 = 31 \cdot 151 \cdot 331$  and  $151 = 31 \cdot 5 - 4$  while  $331 = 31 \cdot 11 - 10$ ;
- :  $1840357 = 43 \cdot 127 \cdot 337$  and  $127 = 43 \cdot 3 - 2$  while  $337 = 43 \cdot 8 - 7$ ;
- :  $12599233 = 97 \cdot 193 \cdot 673$  and  $193 = 97 \cdot 2 - 1$  while  $673 = 97 \cdot 7 - 6$ ;
- :  $13421773 = 53 \cdot 157 \cdot 1613$  and  $157 = 53 \cdot 3 - 2$  while  $1613 = 53 \cdot 31 - 30$ ;
- :  $15162941 = 59 \cdot 233 \cdot 1103$  and  $233 = 59 \cdot 4 - 3$  while  $1103 = 59 \cdot 19 - 18$ ;
- :  $15732721 = 97 \cdot 241 \cdot 673$  and  $97 = 17 \cdot 6 - 5$  while  $241 = 17 \cdot 15 - 14$  also  $673 = 17 \cdot 42 - 41$ ;
- :  $28717483 = 59 \cdot 233 \cdot 2089$  and  $233 = 59 \cdot 4 - 3$  while  $2089 = 59 \cdot 36 - 35$ ;
- :  $29593159 = 43 \cdot 127 \cdot 5419$  and  $127 = 43 \cdot 3 - 2$  while  $5419 = 43 \cdot 129 - 128$ ;
- :  $61377109 = 157 \cdot 313 \cdot 1249$  and  $313 = 157 \cdot 2 - 1$  while  $1249 = 157 \cdot 8 - 7$ ;
- :  $66384121 = 89 \cdot 353 \cdot 2113$  and  $353 = 89 \cdot 4 - 3$  while  $2113 = 89 \cdot 24 - 23$ ;
- :  $67763803 = 103 \cdot 307 \cdot 2143$  and  $307 = 103 \cdot 3 - 2$  while  $2143 = 103 \cdot 21 - 20$ ;
- :  $74658629 = 89 \cdot 397 \cdot 2113$  and  $89 = 23 \cdot 4 - 3$  while  $397 = 23 \cdot 18 - 17$  while  $2113 = 23 \cdot 96 - 95$ ;
- :  $78526729 = 43 \cdot 337 \cdot 5419$  and  $337 = 43 \cdot 8 - 7$  while  $5419 = 43 \cdot 129 - 128$ ;
- :  $90341197 = 103 \cdot 307 \cdot 2857$  and  $307 = 103 \cdot 3 - 2$  while  $2857 = 103 \cdot 28 - 27$ ;
- :  $96916279 = 167 \cdot 499 \cdot 1163$  and  $499 = 167 \cdot 3 - 2$  while  $1163 = 167 \cdot 7 - 6$ ;
- :  $109322501 = 101 \cdot 601 \cdot 1801$  and  $601 = 101 \cdot 6 - 5$  while  $1801 = 101 \cdot 18 - 17$ .

## 16. Observation on the period of the rational number $P/d + d/P$ where $P$ is a 3-Poulet number and $d$ its least prime factor

**Abstract.** In this paper I make the following observation: let  $P$  be a 3-Poulet number,  $d$  its least prime factor and  $q$  one of the other two prime factors; then the length of the period of the rational number  $P/d + d/P$  is for almost any  $P$  equal to  $q - 1$  or equal to  $(q - 1)/n$  or equal to  $(q - 1)*n$ , where  $n$  positive integer.

### Observation:

Let  $P$  be a 3-Poulet number,  $d$  its least prime factor and  $q$  one of the other two prime factors; then the length of the period of the rational number  $P/d + d/P$  is for almost any  $P$  equal to  $q - 1$  or equal to  $(q - 1)/n$  or equal to  $(q - 1)*n$ , where  $n$  positive integer.

### Note:

The sequence of 3-Poulet numbers: 561, 645, 1105, 1729, 1905, 2465, 2821, 4371, 6601, 8481, 8911, 10585, 12801, 13741, 13981, 15841, 16705, 25761, 29341, 30121, 30889, 33153, 34945, 41665, 52633, 57421, 68101, 74665, 83665, 87249, 88561, 91001, 93961, 113201, 115921, 121465, 137149 (...). See the sequence A215672 that I submitted on OEIS.

### Verifying the observation:

(true for 29 from the first 31 such Poulet numbers)

- : for  $P = 561 = 3*11*17$ , the period of  $P/d + d/P$  is equal to 5347593582887700, which has the length  $16 = 17 - 1$ ;
- : for  $P = 645 = 3*5*43$ , the period of  $P/d + d/P$  is equal to 465116279069767441860, which has the length  $21 = (43 - 1)/2$ ;
- : for  $P = 1105 = 5*13*17$ , the period of  $P/d + d/P$  is equal to 452488687782805429864253393665158371040723981900, which has the length  $48 = (17 - 1)*3$ ;
- : for  $P = 1729 = 7*13*19$ , the period of  $P/d + d/P$  is equal to 404858299595141700, which has the length  $18 = 19 - 1$ ;
- : for  $P = 1905 = 3*5*127$ , the period of  $P/d + d/P$  is equal to 157480314960629921259842519685039370078740, which has the length  $42 = (127 - 1)/3$ ;
- : for  $P = 2465 = 5*17*29$ , the period of  $P/d + d/P$  has the length  $112 = (29 - 1)*4$ ;
- : for  $P = 2821 = 7*13*31$ , the period of  $P/d + d/P$  has the length  $30 = 31 - 1$ ;
- : for  $P = 4371 = 3*31*47$ , the period of  $P/d + d/P$  has the length  $690 = (47 - 1)*15$ ;

- : for  $P = 6601 = 7 \cdot 23 \cdot 41$ , the period of  $P/d + d/P$  has the length  $110 = (23 - 1) \cdot 5$ ;
- : for  $P = 8481 = 3 \cdot 11 \cdot 257$ , the period of  $P/d + d/P$  has the length  $256 = 257 - 1$ ;
- : for  $P = 8911 = 7 \cdot 19 \cdot 67$ , the period of  $P/d + d/P$  has the length  $198 = (67 - 1) \cdot 3$ ;
- : for  $P = 10585 = 5 \cdot 29 \cdot 73$ , the period of  $P/d + d/P$  has the length  $56 = (29 - 1) \cdot 2$ ;
- : for  $P = 12801 = 3 \cdot 17 \cdot 251$ , the period of  $P/d + d/P$  has the length  $400 = (17 - 1) \cdot 25$ ;
- : for  $P = 13741 = 7 \cdot 13 \cdot 151$ , the period of  $P/d + d/P$  has the length  $150 = 151 - 1$ ;
- : for  $P = 13981 = 11 \cdot 31 \cdot 41$ , the period of  $P/d + d/P$  has the length  $15 = (31 - 1)/2$ ;
- : for  $P = 15841 = 7 \cdot 31 \cdot 73$ , the period of  $P/d + d/P$  has the length  $120 = (31 - 1) \cdot 4$ ;
- : for  $P = 16705 = 5 \cdot 13 \cdot 257$ , the period of  $P/d + d/P$  has the length  $768 = (257 - 1) \cdot 3$ ;
- : for  $P = 29341 = 13 \cdot 37 \cdot 61$ , the period of  $P/d + d/P$  has the length  $60 = 61 - 1$ ;
- : for  $P = 30121 = 7 \cdot 13 \cdot 331$ , the period of  $P/d + d/P$  has the length  $330 = 331 - 1$ ;
- : for  $P = 30889 = 17 \cdot 23 \cdot 79$ , the period of  $P/d + d/P$  has the length  $286 = (23 - 1) \cdot 13$ ;
- : for  $P = 33153 = 3 \cdot 43 \cdot 257$ , the period of  $P/d + d/P$  has the length  $5376 = (257 - 1) \cdot 21$ ;
- : for  $P = 34945 = 5 \cdot 29 \cdot 241$ , the period of  $P/d + d/P$  has the length  $420 = (29 - 1) \cdot 15$ ;
- : for  $P = 41665 = 5 \cdot 13 \cdot 641$ , the period of  $P/d + d/P$  has the length  $96 = (13 - 1) \cdot 8$ ;
- : for  $P = 57421 = 7 \cdot 13 \cdot 631$ , the period of  $P/d + d/P$  has the length  $630 = 631 - 1$ ;
- : for  $P = 68101 = 11 \cdot 41 \cdot 151$ , the period of  $P/d + d/P$  has the length  $75 = (151 - 1)/2$ ;
- : for  $P = 74665 = 5 \cdot 109 \cdot 137$ , the period of  $P/d + d/P$  has the length  $216 = (109 - 1) \cdot 2$ ;
- : for  $P = 83665 = 5 \cdot 29 \cdot 577$ , the period of  $P/d + d/P$  has the length  $4032 = (577 - 1) \cdot 7$ ;
- : for  $P = 87249 = 3 \cdot 127 \cdot 229$ , the period of  $P/d + d/P$  has the length  $1596 = (229 - 1) \cdot 7$ ;
- : for  $P = 88561 = 11 \cdot 83 \cdot 97$ , the period of  $P/d + d/P$  has the length  $3936 = (97 - 1) \cdot 41$ .

**Exceptions:**

- : for  $P = 25761 = 3 \cdot 31 \cdot 277$ , the period of  $P/d + d/P$  has the length 345;
- : for  $P = 52633 = 7 \cdot 73 \cdot 103$ , the period of  $P/d + d/P$  has the length 136.

## 17. Conjecture on the numbers $6(m + n) + 1$ where $m$ and $n$ are the two prime factors of a 2-Poulet number

**Abstract.** In this paper I make the following conjecture: there exist an infinity of 2-Poulet numbers  $P = m \cdot n$  having the property that the number  $q = 6(m + n) + 1$  is prime.

### Conjecture 1:

There exist an infinity of 2-Poulet numbers  $P = m \cdot n$  having the property that the number  $q = 6(m + n) + 1$  is prime.

The sequence of 2-Poulet numbers (A214305 in OEIS):

: 341, 1387, 2047, 2701, 3277, 4033, 4369, 4681, 5461, 7957, 8321, 10261, 13747, 14491, 15709, 18721, 19951, 23377, 31417, 31609, 31621, 35333, 42799, 49141, 49981, 60701, 60787, 65077, 65281, 80581, 83333, 85489, 88357, 90751, 104653, 123251, 129889, 130561, 150851, 162193, 164737, 181901, 188057, 194221, 196093, 215749, 219781, 220729 (...)

### The sequence of primes $q$ :

:  $q = 673$  for  $P = 2047 = 23 \cdot 89$ ;  
:  $q = 661$  for  $P = 2701 = 37 \cdot 73$ ;  
:  $q = 853$  for  $P = 3277 = 29 \cdot 113$ ;  
:  $q = 877$  for  $P = 4033 = 37 \cdot 109$ ;  
:  $q = 1093$  for  $P = 4681 = 31 \cdot 151$ ;  
:  $q = 1021$  for  $P = 5461 = 43 \cdot 127$ ;  
:  $q = 1093$  for  $P = 7957 = 73 \cdot 109$ ;  
:  $q = 1753$  for  $P = 13747 = 59 \cdot 233$ ;  
:  $q = 2281$  for  $P = 14491 = 43 \cdot 337$ ;  
:  $q = 1741$  for  $P = 18721 = 97 \cdot 193$ ;  
:  $q = 2113$  for  $P = 19951 = 71 \cdot 281$ ;  
:  $q = 2029$  for  $P = 23377 = 97 \cdot 241$ ;  
:  $q = 3037$  for  $P = 31609 = 73 \cdot 433$ ;  
:  $q = 2917$  for  $P = 35333 = 89 \cdot 397$ ;  
:  $q = 4621$  for  $P = 65281 = 97 \cdot 673$ ;  
:  $q = 8293$  for  $P = 80581 = 61 \cdot 1321$ ;  
:  $q = 4513$  for  $P = 90751 = 151 \cdot 601$ ;  
:  $q = 2889$  for  $P = 123251 = 59 \cdot 2089$ ;  
:  $q = 5197$  for  $P = 129889 = 193 \cdot 673$ ;  
:  $q = 5113$  for  $P = 150851 = 251 \cdot 601$ ;  
:  $q = 13477$  for  $P = 220729 = 103 \cdot 2143$ ;  
(...)  
:  $q = 2112493$  for  $P = 27547091281 = 117361 \cdot 234721$ ;  
:  $q = 2581933$  for  $P = 27547169521 = 78241 \cdot 352081$ ;  
:  $q = 2012401$  for  $P = 27549352151 = 143743 \cdot 191657$ ;  
:  $q = 20127253$  for  $P = 27550161997 = 8233 \cdot 3346309$ ;  
:  $q = 14751637$  for  $P = 27603401233 = 11279 \cdot 2447327$ ;

:  $q = 7326733$  for  $P = 27604984321 = 23041 * 1198081$ ;  
 :  $q = 4681921$  for  $P = 27615119599 = 37159 * 743161$ ;  
 :  $q = 3324613$  for  $P = 27632967001 = 55411 * 498691$ ;  
 :  $q = 2421733$  for  $P = 27642607621 = 87421 * 316201$ ;  
 :  $q = 2240317$  for  $P = 27653056649 = 101833 * 271553$ ;  
 :  $q = 4017697$  for  $P = 27685810639 = 44273 * 625343$ ;  
 :  $q = 2038693$  for  $P = 27708447397 = 135913 * 203869$ ;  
 :  $q = 2680141$  for  $P = 27712970209 = 74449 * 372241$ ;  
 (...)

Note the chain of four consecutive 2-Poulet numbers 27547091281, 27547169521, 27549352151, 27550161997 for which  $q$  is prime.

## Part Three.

### Prime producing quadratic polynomials

#### 1. A list of known root prime-generating quadratic polynomials producing more than 23 distinct primes in a row

**Abstract.** A simple list of known such polynomials, indexed by the value of discriminants, containing no analysis but the introduction of the “root prime generating polynomial” notion.

I listed below the polynomials (after the value of discriminant). In the brackets we have the polynomials that generate same primes but in reverse order (any prime-producing polynomial has such a reversal). The list contains 42 polynomials (84 with their reversals). I didn't consider redundant primes to not complicate the list furthermore. I discovered myself all the polynomials with the font italic (32(64)). I know the other ones from the articles available on Internet like *Prime-Generating Polynomial* from Wolfram Math World or sites like Rivera's *The Prime Puzzles & Problems Connection*.

Note: because of the special nature of the number 1, I considered the polynomials that generate that number too as prime-generating polynomials, but, for the purists, I indexed with specification “d.p.” distinct primes and “d.p.1.” distinct primes plus number 1 (in absolute value).

Note: a “**root prime-generating polynomial**” I consider to be the prime-generating polynomial that has two properties:

(1) for  $n = -1$  gives a non-prime term (for instance,  $8n^2 + 88n + 43$  is not a root prime-generating polynomial because for  $n = -1$  we have the prime term (in absolute value)  $-37$  and for  $n = n - 39$  we have the “complete” root prime-generating polynomial:  $8n^2 - 488n + 7243$ );

(2) there is no other prime-generating polynomial with the same value of discriminant that generates the same amount of primes in a row, having coefficients of smaller values.

Note: I submitted few of these polynomials to OEIS.

Discriminant equal to  $-222643$ :

35 d.p.:  $43n^2 - 537n + 2971$  ( $43n^2 - 2387n + 34421$ ).

Discriminant equal to  $-23472$ :

26 d.p.:  $36n^2 - 408n + 1319$  ( $36n^2 - 1392n + 13619$ ).

Discriminant equal to  $-13203$ :

28 d.p.:  $81n^2 - 1323n + 5443$  ( $81n^2 - 3051n + 28771$ );

25 d.p.:  $9n^2 - 219n + 1699$  ( $9n^2 - 213n + 1627$ ).

Discriminant equal to  $-10432$ :

23 d.p.:  $64n^2 - 1192n + 5591$  ( $64n^2 - 1624n + 10343$ ).

Discriminant equal to -8523:

23 d.p.:  $27n^2 - 489n + 2293$  ( $27n^2 - 699n + 4603$ ).

Discriminant equal to -7987:

23 d.p.:  $49n^2 - 469n + 1163$  ( $49n^2 - 1687n + 14561$ ).

Discriminant equal to -4075:

32 d.p.:  $25n^2 - 365n + 1373$  ( $25n^2 - 1185n + 14083$ ).

Discriminant equal to -2608:

31 d.p.:  $16n^2 - 292n + 1373$  ( $16n^2 - 668n + 7013$ );

30 d.p.:  $16n^2 - 300n + 1447$  ( $16n^2 - 628n + 6203$ ).

Discriminant equal to -1467:

40 d.p.:  $9n^2 - 231n + 1523$  ( $9n^2 - 471n + 6203$ ).

Discriminant equal to -708:

29 d.p.:  $6n^2 + 6n + 31$  ( $6n^2 - 342n + 4903$ ).

Discriminant equal to -652:

40 d.p.:  $4n^2 - 154n + 1523$  ( $4n^2 - 158n + 1601$ ).

Discriminant equal to -232:

29 d.p.:  $2n^2 + 29$  ( $2n^2 - 112n + 1597$ ).

Discriminant equal to -163:

40 d.p.:  $n^2 + n + 41$  ( $n^2 - 79n + 1601$ ).

Discriminant equal to 293:

24 d.p.1.:  $n^2 + n - 73$  ( $n^2 - 47n + 479$ ).

Discriminant equal to 437:

28 d.p.1.:  $n^2 + n - 109$  ( $n^2 + 55n + 647$ ).

Discriminant equal to 677:

25 d.p.1.:  $13n^2 - 313n + 1871$  ( $13n^2 - 311n + 1847$ );

23 d.p.:  $n^2 + 3n - 167$  ( $n^2 - 49n + 431$ ).

Discriminant equal to 1077:

24 d.p.1.:  $3n^2 + 3n - 89$  ( $3n^2 - 141n + 1567$ ).

Discriminant equal to 1172:

29 d.p.1.:  $4n^2 - 90n + 433$  ( $4n^2 - 142n + 1187$ ).

Discriminant equal to 1253:

27 d.p.1.:  $7n^2 + 7n - 43$  ( $7n^2 - 371n + 4871$ ).

Discriminant equal to 1592:

28 d.p.1.:  $2n^2 - 199$  ( $2n^2 + 108n + 1259$ ).

Discriminant equal to 6368:

31 d.p.:  $8n^2 + 8n - 197$  ( $8n^2 - 488n + 7243$ ).

Discriminant equal to 19808:

23 d.p.:  $104n^2 - 2200n + 11587$  ( $104n^2 - 2376n + 13523$ ).

Discriminant equal to 25472:

35 d.p.:  $4n^2 + 12n - 1583$  ( $4n^2 - 284n + 3449$ );

31 d.p.:  $32n^2 - 944n + 6763$  ( $32n^2 - 976n + 7243$ );

29 d.p.:  $16n^2 - 408n + 2203$  ( $16n^2 - 488n + 3323$ ).

Discriminant equal to 57312:

35 d.p.:  $72n^2 - 1416n + 6763$  ( $72n^2 - 1752n + 10459$ ).

Discriminant equal to 64917:

35 d.p.l.:  $27n^2 - 741n + 4483$  ( $27n^2 - 1095n + 10501$ );

33 d.p.:  $81n^2 - 2247n + 15383$  ( $81n^2 - 2937n + 26423$ );

32 d.p.:  $27n^2 - 753n + 4649$  ( $27n^2 - 921n + 7253$ );

24 d.p.:  $9n^2 + 9n - 1801$  ( $9n^2 - 423n + 3167$ ).

Discriminant equal to 78008:

28 d.p.:  $98n^2 - 2128n + 11353$  ( $98n^2 - 3164n + 25339$ ).

Discriminant equal to 101888:

31 d.p.:  $4n^2 - 428n + 5081$  ( $4n^2 + 188n - 4159$ );

24 d.p.l.:  $128n^2 - 1216n + 2689$  ( $128n^2 - 4672n + 42433$ );

Discriminant equal to 159200:

27 d.p.:  $100n^2 - 2820n + 19483$  ( $4n^2 - 2380n + 13763$ ).

Discriminant equal to 259668:

45 d.p.:  $36n^2 - 810n + 2753$  ( $36n^2 - 2358n + 36809$ );

24 d.p.:  $108n^2 - 2130n + 9901$  ( $108n^2 - 2838n + 18043$ ).

Discriminant equal to 979373:

43 d.p.:  $47n^2 - 1701n + 10181$  ( $47n^2 - 2247n + 21647$ ).

Discriminant equal to 1038672:

29 d.p.:  $144n^2 - 2196n + 6569$  ( $144n^2 - 5868n + 57977$ ).

Discriminant equal to 1398053:

43 d.p.:  $103n^2 - 4707n + 50383$  ( $103n^2 - 3945n + 34381$ ).

I also submit the following problem: find a value of discriminant, beside the ones from the following list: -222643, -4075, -2608, -1467, -652, -163, 6368, 25472, 57312, 64917, 101888, 259668, 979373, 1398053, for which a quadratic polynomial having this discriminant generates 30 or more distinct primes in a row.

I list below the polynomials that I know that generates 30 or more distinct primes in a row (in the brackets are the reverse polynomials, that generates same primes in reverse order):

$43n^2 - 537n + 2971$  ( $43n^2 - 2387n + 34421$ );  
 $9n^2 - 231n + 1523$  ( $9n^2 - 471n + 6203$ );  
 $4n^2 - 154n + 1523$  ( $4n^2 - 158n + 1601$ );  
 $n^2 + n + 41$  ( $n^2 - 79n + 1601$ );  
 $8n^2 + 8n - 197$  ( $8n^2 - 488n + 7243$ );  
 $36n^2 - 810n + 2753$  ( $36n^2 - 2358n + 36809$ );  
 $47n^2 - 1701n + 10181$  ( $47n^2 - 2247n + 21647$ );  
 $103n^2 - 4707n + 50383$  ( $103n^2 - 3945n + 34381$ ).

I list below the polynomials that I discovered myself that generates 30 or more distinct primes in a row (few of them are posted on OEIS):

$25n^2 - 365n + 1373$  ( $25n^2 - 1185n + 14083$ );  
 $16n^2 - 292n + 1373$  ( $16n^2 - 668n + 7013$ );  
 $16n^2 - 300n + 1447$  ( $16n^2 - 628n + 6203$ );  
 $4n^2 + 12n - 1583$  ( $4n^2 - 284n + 3449$ );  
 $32n^2 - 944n + 6763$  ( $32n^2 - 976n + 7243$ );  
 $72n^2 - 1416n + 6763$  ( $72n^2 - 1752n + 10459$ );  
 $81n^2 - 2247n + 15383$  ( $81n^2 - 2937n + 26423$ );  
 $27n^2 - 753n + 4649$  ( $27n^2 - 921n + 7253$ );  
 $4n^2 - 428n + 5081$  ( $4n^2 + 188n - 4159$ ).

## 2. Ten prime-generating quadratic polynomials

**Abstract.** In two of my previous papers I treated quadratic polynomials which have the property to produce many primes in a row: in one of them I listed forty-two such polynomials which generate more than twenty-three primes in a row and in another one I listed few generic formulas which may conduct to find such prime-producing quadratic polynomials. In this paper I will present ten such polynomials which I discovered and posted in OEIS, each accompanied by its first fifty terms and some comments about it.

### I.

The polynomial  $16n^2 - 300n + 1447$ .

Its first fifty terms:

1447, 1163, 911, 691, 503, 347, 223, 131, 71, 43, 47, 83, 151, 251, 383, 547, 743, 971, 1231, 1523, 1847, 2203, 2591, 3011, 3463, 3947, 4463, 5011, 5591, 6203, 6847, 7523, 8231, 8971, 9743, 10547, 11383, 12251, 13151, 14083, 15047, 16043, 17071, 18131, 19223, 20347, 21503, 22691, 23911, 25163, 26447.

Comments:

This polynomial generates 30 primes in a row starting from  $n = 0$ .

The polynomial  $16n^2 - 628n + 6203$  generates the same primes in reverse order.

I found in the same family of prime-generating polynomials (with the discriminant equal to  $-163 \cdot 2^p$ , where  $p$  is even), the polynomials  $4n^2 - 152n + 1607$ , generating 40 primes in row starting from  $n = 0$  (20 distinct ones) and  $4n^2 - 140n + 1877$ , generating 36 primes in row starting from  $n = 0$  (18 distinct ones).

The following 5 (10 with their "reversal" polynomials) are the only ones I know from the family of Euler's polynomial  $n^2 + n + 41$  (having their discriminant equal to a multiple of  $-163$ ) that generate more than 30 distinct primes in a row starting from  $n = 0$  (beside the Escott's polynomial  $n^2 - 79n + 1601$ ):

- (1)  $4n^2 - 154n + 1523$  ( $4n^2 - 158n + 1601$ );
- (2)  $9n^2 - 231n + 1523$  ( $9n^2 - 471n + 6203$ );
- (3)  $16n^2 - 292n + 1373$  ( $16n^2 - 668n + 7013$ );
- (4)  $25n^2 - 365n + 1373$  ( $25n^2 - 1185n + 14083$ );
- (5)  $16n^2 - 300n + 1447$  ( $16n^2 - 628n + 6203$ ).

### II.

The polynomial  $2n^2 - 108n + 1259$ .

Its first fifty terms:

1259, 1153, 1051, 953, 859, 769, 683, 601, 523, 449, 379, 313, 251, 193, 139, 89, 43, 1, -37, -71, -101, -127, -149, -167, -181, -191, -197, -199, -197, -191, -181, -167, -149, -127, -101, -71, -37, 1, 43, 89, 139, 193, 251, 313, 379, 449, 523, 601, 683, 769.

Comments:

This polynomial generates 92 primes (66 distinct ones) for  $n$  from 0 to 99 (in fact the next two terms are still primes but we keep the range 0-99, customary for comparisons), just three primes less than the record held by the Euler's polynomial for  $n = m - 35$ , which is  $m^2 - 69m + 1231$ , but having six distinct primes more than this one.

The non-prime terms in the first 100 are: 1 (taken twice),  $1369 = 37^2$ ,  $1849 = 43^2$ ,  $4033 = 37 \cdot 109$ ,  $5633 = 43 \cdot 131$ ,  $7739 = 71 \cdot 109$  and  $8251 = 37 \cdot 223$ .

For  $n = 2m - 34$  we obtain the polynomial  $8m^2 - 488m + 7243$ , which generates 31 primes in a row starting from  $m = 0$ .

For  $n = 4m - 34$  we obtain the polynomial  $32m^2 - 976m + 7243$ , which generates 31 primes in row starting from  $m = 0$ .

### III.

The polynomial  $2n^2 - 212n + 5419$ .

Its first fifty terms:

5419, 5209, 5003, 4801, 4603, 4409, 4219, 4033, 3851, 3673, 3499, 3329, 3163, 3001, 2843, 2689, 2539, 2393, 2251, 2113, 1979, 1849, 1723, 1601, 1483, 1369, 1259, 1153, 1051, 953, 859, 769, 683, 601, 523, 449, 379, 313, 251, 193, 139, 89, 43, 1, -37, -71, -101, -127, -149, -167, -181.

Comments:

This polynomial generates 92 primes (57 distinct ones) for  $n$  from 0 to 99 (in fact the next seven terms are still primes but we keep the range 0-99, customary for comparisons), just three primes less than the record held by the Euler's polynomial for  $n = m - 35$ , which is  $m^2 - 69m + 1231$ .

The non-prime terms in the first 100 are: 1,  $1369 = 37^2$ ,  $1849 = 43^2$ ,  $4033 = 37 \cdot 109$  (all taken twice).

For  $n = 2m + 54$  we obtain the polynomial  $8m^2 + 8m - 197$ , which generates 31 primes in a row starting from  $m = 0$  (the polynomial  $8m^2 - 488m + 7243$  generates the same 31 primes, but in reverse order).

### IV.

The polynomial  $25n^2 - 1185n + 14083$ .

Its first fifty terms:

14083, 12923, 11813, 10753, 9743, 8783, 7873, 7013, 6203, 5443, 4733, 4073, 3463, 2903, 2393, 1933, 1523, 1163, 853, 593, 383, 223, 113, 53, 43, 83, 173, 313, 503, 743, 1033, 1373, 1763, 2203, 2693, 3233, 3823, 4463, 5153, 5893, 6683, 7523, 8413, 9353, 10343, 12473, 13613, 14803, 16043, 17333.

Comments:

The polynomial generates 32 primes in row starting from  $n = 0$ .

The polynomial  $25n^2 - 365n + 1373$  generates the same primes in reverse order.

This family of prime-generating polynomials (with the discriminant equal to  $-4075 = -163 \cdot 5^2$ ) is interesting for generating primes of same form: the polynomial  $25n^2 - 395(n + 1601)$  generates 16 primes of the form  $10k + 1$  (1601, 1231, 911, 641, 421, 251, 131, 61, 41, 71, 151, 281, 461, 691, 971, 1301) and the polynomial  $25n^2 + 25n + 47$  generates 16 primes of the form  $10k + 7$  (47, 97, 197, 347, 547, 797, 1097, 1447, 1847, 2297, 2797, 3347, 3947, 4597, 5297, 6047).

Note:

All the polynomials of the form  $25n^2 + 5n + 41$ ,  $25n^2 + 15n + 43, \dots, 25n^2 + 5(2k + 1)n + p, \dots, 25n^2 + 5 \cdot 79n + 1601$ , where  $p$  is a (prime) term of the Euler's polynomial  $p = k^2 + k + 41$ , from  $k = 0$  to  $k = 39$ , have their discriminant equal to  $-4075 = -163 \cdot 5^2$ .

## V.

The polynomial  $16n^2 - 292n + 1373$ .

Its first fifty terms:

1373, 1097, 853, 641, 461, 313, 197, 113, 61, 41, 53, 97, 173, 281, 421, 593, 797, 1033, 1301, 1601, 1933, 2297, 2693, 3121, 3581, 4073, 4597, 5153, 5741, 6361, 7013, 7697, 8413, 9161, 9941, 10753, 11597, 12473, 13381, 14321, 15293, 16297, 17333, 18401, 20633, 21797, 22993, 24221, 25481, 26773.

Comments:

The polynomial generates 31 primes in row starting from  $n = 0$ .

The polynomial  $16n^2 - 668n + 7013$  generates the same primes in reverse order.

Note:

All the polynomials of the form  $p^2n^2 \pm p^*n + 41$ ,  $p^2n^2 \pm 3p^*n + 43$ ,  $p^2n^2 \pm 5p^*n + 47, \dots, p^2n^2 \pm (2k+1)p^*n + q, \dots, p^2n^2 \pm 79p^*n + 1601$ , where  $q$  is a (prime) term of the Euler's polynomial  $q = k^2 + k + 41$ , from  $k = 0$  to  $k = 39$ , have their discriminant equal to  $-163p^2$ ; the demonstration is easy: the discriminant is equal to  $b^2 - 4ac = (2k + 1)^2p^2 - 4q^*p^2 = -p^2((2k + 1)^2 - 4q) = -p^2(4k^2 + 4k + 1 - 4k^2 - 4k - 164) = -163p^2$ .

Observation:

Many of the polynomials formed this way have the capacity to generate many primes in row.

Examples:

- :  $9n^2 + 3n + 41$  generates 27 primes in row starting from  $n = 0$  (and 40 primes for  $n = n - 13$ );
- :  $9n^2 - 237n + 1601$  generates 27 primes in row starting from  $n = 0$ ;
- :  $16n^2 + 4n + 41$  generates, for  $n = n - 21$  (that is  $16n^2 - 668n + 7013$ ) 31 primes in row.

## VI.

The polynomial  $4n^2 - 284n + 3449$ .

Its first fifty terms:

3449, 3169, 2897, 2633, 2377, 2129, 1889, 1657, 1433, 1217, 1009, 809, 617, 433, 257, 89, -71, -223, -367, -503, -631, -751, -863, -967, -1063, -1151, -1231, -1303, -1367, -1423, -1471, -1511, -1543, -1567, -1583, -1591, -1591, -1583, -1567, -1543, -1511, -1471, -1367, -1303, -1231, -1151, -1063, -967, -863, -751.

Comments:

The polynomial successively generates 35 primes or negative values of primes starting at  $n = 0$ .

This polynomial generates 95 primes in absolute value (60 distinct ones) for  $n$  from 0 to 99, equaling the record held by the Euler's polynomial for  $n = m - 35$ , which is  $m^2 - 69m + 1231$ .

The non-prime terms (in absolute value) up to  $n = 99$  are:  $1591 = 37 \cdot 43$ ,  $3737 = 37 \cdot 101$ ,  $4033 = 37 \cdot 109$ ;  $5633 = 43 \cdot 131$ ;  $5977 = 43 \cdot 139$ ;  $9017 = 71 \cdot 127$ .

The polynomial  $4n^2 + 12n - 1583$  generates the same 35 primes in row starting from  $n = 0$  in reverse order.

Note:

In the same family of prime-generating polynomials (with the discriminant equal to  $199 \cdot 2^p$ , where  $p$  is odd) there are the polynomial  $32n^2 - 944n + 6763$  (with its "reversed polynomial"  $32m^2 - 976m + 7243$ , for  $m = 30 - n$ ), generating 31 primes in row, and the polynomial  $4n^2 - 428n + 5081$  (with  $4m^2 + 188m - 4159$ , for  $m = 30 - n$ ), generating 31 primes in row.

## VII.

The polynomial  $n^2 + 3n - 167$ .

Its first fifty terms:

-167, -163, -157, -149, -139, -127, -113, -97, -79, -59, -37, -13, 13, 41, 71, 103, 137, 173, 211, 251, 293, 337, 383, 431, 481, 533, 587, 643, 701, 761, 823, 887, 953, 1021, 1091, 1163, 1237, 1313, 1391, 1471, 1553, 1637, 1723, 1811, 1901, 1993, 2087, 2183, 2381, 2483.

Comments:

The polynomial generates 24 primes in absolute value (23 distinct ones) in row starting from  $n = 0$  (and 42 primes in absolute value for  $n$  from 0 to 46).

The polynomial  $n^2 - 49n + 431$  generates the same primes in reverse order.

Note:

We found in the same family of prime-generating polynomials (with the discriminant equal to 677) the polynomial  $13n^2 - 311n + 1847$  ( $13n^2 - 469n + 4217$ ) generating 23 primes and two noncomposite numbers (in absolute value) in row starting from  $n = 0$  (1847, 1549, 1277,

1031, 811, 617, 449, 307, 191, 101, 37, -1, -13, 1, 41, 107, 199, 317, 461, 631, 827, 1049, 1297, 1571, 1871).

Note:

Another interesting algorithm to produce prime-generating polynomials could be  $N = m*n^2 + (6*m + 1)*n + 8*m + 3$ , where  $m$ ,  $6*m + 1$  and  $8*m + 3$  are primes. For  $m = 7$  then  $n = t - 20$  we get  $N = 7*t^2 - 237*t + 1999$ , which generates the following primes: 239, 163, 101, 53, 19, -1, -7, 1, 23, 59, 109, 173, 251 (we can see the same pattern: ..., -1, -m, 1, ...).

### VIII.

The polynomial  $81*n^2 - 2247*n + 15383$ .

Its first forty terms:

15383, 13217, 11213, 9371, 7691, 6173, 4817, 3623, 2591, 1721, 1013, 467, 83, -139, -199, -97, 167, 593, 1181, 1931, 2843, 3917, 5153, 6551, 8111, 9833, 11717, 13763, 15971, 18341, 20873, 23567, 26423, 29441, 32621, 35963, 39467, 43133, 46961, 50951.

Comments:

The polynomial generates 33 primes/negative values of primes in row starting from  $n = 0$ .

The polynomial  $81*n^2 - 2937*n + 26423$  generates the same primes in reverse order.

Note:

We found in the same family of prime-generating polynomials (with the discriminant equal to  $64917 = 3^2*7213$ ) the polynomial  $27*n^2 - 753*n + 4649$  (with its "reversed polynomial"  $27*n^2 - 921*n + 7253$ ), generating 32 primes in row and the polynomial  $27*n^2 - 741*n + 4483$  ( $27*n^2 - 1095*n + 10501$ ), generating 35 primes in row, if we consider that 1 is prime (which seems to be constructive in the study of prime-generating polynomials, at least).

Note:

The polynomial  $36*n^2 - 810*n + 2753$ , which is the known quadratic polynom that generates the most distinct primes in row (45), has the discriminant equal to  $259668 = 2^2*3^2*7213$ .

### IX.

The polynomial  $4*n^2 + 12*n - 1583$ .

Its first forty terms:

-1583, -1567, -1543, -1511, -1471, -1423, -1367, -1303, -1231, -1151, -1063, -967, -863, -751, -631, -503, -367, -223, -71, 89, 257, 433, 617, 809, 1009, 1217, 1433, 1657, 1889, 2129, 2377, 2633, 2897, 3169, 3449, 3737, 4033, 4337, 4649, 4969, 14561, 14083, 13613, 13151, 12697, 12251, 11813, 11383, 10961, 10547, 10141, 9743, 9353, 8971, 8597, 8231, 7873, 7523, 7181, 6847, 6521, 6203, 5893, 5591, 5297, 5011, 4733, 4463, 4201, 3947, 3701, 3463, 3233, 3011, 2797, 2591, 2393, 2203, 2021, 1847.

Comments:

The polynomial generates 35 primes/negative values of primes in row starting from  $n = 0$ .

The polynomial  $4n^2 - 284n + 3449$  generates the same primes in reverse order.

Other related polynomials are:

- : for  $n = 6n + 6$  then  $n = n - 11$  we get  $144n^2 - 2808n + 12097$  which generates 16 primes in a row starting from  $n = 0$  (with the discriminant equal to  $2^9 \cdot 3^2 \cdot 199$ );
- : for  $n = 12n + 12$  then  $n = n - 15$  we get  $576n^2 - 15984n + 109297$  which generates 17 primes in a row starting from  $n = 0$  (with the discriminant equal to  $2^{11} \cdot 3^2 \cdot 199$ ).

Note: so this polynomials opens at least two directions of study:

- (1) polynomials of type  $4n^2 + 12n - p$ , where  $p$  is prime (could be of the form  $30k + 23$ );
- (2) polynomials with the discriminant equal to  $2^n \cdot 3^m \cdot 199$ , where  $n$  is odd and  $m$  is even (an example of such polynomial, with the discriminant equal to  $2^5 \cdot 3^4 \cdot 199$  is  $36n^2 - 1020n + 3643$  which generates 32 primes for values from 0 to 34).

## X.

The polynomial  $4n^2 - 482n + 14561$ .

Its first forty terms:

14561, 14083, 13613, 13151, 12697, 12251, 11813, 11383, 10961, 10547, 10141, 9743, 9353, 8971, 8597, 8231, 7873, 7523, 7181, 6847, 6521, 6203, 5893, 5591, 5297, 5011, 4733, 4463, 4201, 3947, 3701, 3463, 3233, 3011, 2797, 2591, 2393, 2203, 2021, 1847.

Comments:

This polynomial generates 88 distinct primes for  $n$  from 0 to 99, just two primes less than the record held by the polynomial discovered by N. Boston and M. L. Greenwood, that is  $41n^2 - 4641n + 88007$  (this polynomial is sometimes cited as  $41n^2 + 33n - 43321$ , which is the same for the input values  $[-57, 42]$ ).

Note:

The non-prime terms in the first 100 are:  $10961 = 97 \cdot 113$ ;  $10547 = 53 \cdot 199$ ;  $9353 = 47 \cdot 199$ ;  $7181 = 43 \cdot 167$ ;  $6847 = 41 \cdot 167$ ;  $5893 = 71 \cdot 83$ ;  $3233 = 53 \cdot 61$ ;  $2021 = 43 \cdot 47$ ;  $1681 = 41^2$ ;  $1763 = 41 \cdot 43$ ;  $2491 = 47 \cdot 53$ ;  $4331 = 61 \cdot 71$ .

Note:

For  $n = m + 41$  we obtain the polynomial  $4m^2 - 154m + 1523$ , which generates 40 primes in a row starting from  $m = 0$ .

### 3. Seventeen generic formulas that may generate prime-producing quadratic polynomials

**Abstract.** In one of my previous papers I listed forty-two quadratic polynomials which generate more than twenty-three primes in a row, from which ten were already known from the articles available on Internet and thirty-two were discovered by me. In this paper I list few generic formulas which may conduct to find such prime-producing quadratic polynomials.

#### I.

The formula  $8n^2 + (2p + 2)n + p$ , where  $p$  is prime.

Examples:

- : for  $p = 43$  we have the polynomial  $8n^2 + 88n + 43$  which generates 26 distinct primes for values of  $n$  from 0 to 25; also, for  $m = n - 39$  is obtained the root prime-generating polynomial  $8m^2 - 488m + 7243$  which generates, from values of  $m$  from 0 to 30, thirty-one distinct primes in a row;
- : for  $p = 29$  we have the polynomial  $8n^2 + 60n + 29$  which generates 20 distinct primes or squares of primes for values of  $n$  from 0 to 19;
- : for  $p = 19$  we have the polynomial  $8n^2 + 40n + 19$  which generates 20 distinct primes for values of  $m$  from 0 to 19, where  $m = n - 12$ , in other words from this polynomial is obtained the root prime-generating polynomial  $8m^2 - 152m + 691$ .

#### II.

The formula  $2m^2n^2 + 40mn + 1$ , where  $m$  is positive integer.

Examples:

- : for  $m = 1$  we have the polynomial  $2n^2 + 40n + 1$  which generates 36 distinct primes or squares of primes for values of  $n$  from 0 to 35; also, for  $m = 6n + 1$ , is obtained the polynomial  $72m^2 + 264m + 43$  which generates 9 distinct primes in a row; for  $m = 7n + 5$  is obtained the polynomial  $98m^2 + 420m + 251$  which generates 14 distinct primes in a row; for  $m = 8n + 6$  is obtained the polynomial  $128m^2 + 512m + 313$  which generates 13 distinct primes or squares of primes in a row;
- : for  $m = 8$  we have the polynomial  $128n^2 + 320n + 1$  which generates 17 distinct primes in a row for values of  $n$  from 0 to 16.

#### III.

The formula  $2m^2n^2 - 199$ , where  $m$  is positive integer.

Examples:

- : for  $m = 1$  we have the polynomial  $2*n^2 - 199$  which generates 28 distinct primes in a row for values of  $n$  from 0 to 27; also, for  $m = 2*n + 29$ , is obtained the polynomial  $8*m^2 + 232*m + 1483$  which generates 31 distinct primes respectively 62 redundant primes in a row; also, for  $m = 2*n - 1$ , is obtained the polynomial  $8*m^2 - 8*m - 197$  which generates 31 distinct primes in a row;
- : for  $m = 2$  we have the polynomial  $8*n^2 - 199$  which generates 14 distinct primes in a row; also, for  $m = n - 13$  we have the polynomial  $8*m^2 - 208*m + 1153$  which generates 31 distinct primes and 44 redundant primes in a row;
- : for  $m = 3$  we have the polynomial  $18*n^2 - 199$  which generates 18 distinct primes in a row;
- : for  $m = 4$  we have the polynomial  $32*n^2 - 199$  which generates 27 distinct primes or squares of primes in a row.

#### IV.

The formula  $2*m^2*n^2 + 29$ , where  $m$  is positive integer.

Examples:

- : for  $m = 1$  we have the Sierpinski's polynomial  $2*n^2 + 29$  which generates 29 distinct primes in a row;
- : for  $m = 2$  we have the polynomial  $8*n^2 + 29$  which generates 15 distinct primes in a row.

#### V.

The formula  $m^2*n^2 + m*n + 41$ , where  $m$  is positive integer.

Examples:

- : for  $m = 1$  we have the Euler's polynomial  $n^2 + n + 41$  which generates 40 distinct primes in a row;
- : for  $m = 2$  we have the polynomial  $4*n^2 + 2*n + 41$  which generates 20 distinct primes in a row; also, for  $m = 2*n + 1$  is obtained the polynomial  $16*m^2 + 20*m + 47$  which generates 20 distinct primes in a row; also for  $t = t - 10$  we have the polynomial  $16*t^2 - 300*t + 1447$  which generates 31 primes in a row;
- : for  $m = 3$  we have the polynomial  $9*n^2 + 3*n + 41$  which generates 27 distinct primes in a row; also, for  $m = n - 13$  is obtained the polynomial  $9*n^2 - 231*n + 1523$  which generates 40 distinct primes in a row.

#### VI.

The formula  $m^2*n^2 + 2*m*n + 59$ , where  $m$  is positive integer.

Examples:

- : for  $m = 2$  we have the polynomial  $4*n^2 + 4*n + 59$  which generates 14 distinct primes in a row;
- : for  $m = 6$  we have the polynomial  $36*n^2 + 12*n + 59$  which generates 15 distinct primes in a row; also for  $m = n - 4$  is obtained the polynomial  $36*m^2 - 276*m + 587$  which generates 19 distinct primes in a row;
- : for  $m = 12$  we have the polynomial  $144*n^2 + 24*n + 59$  which generates 12 distinct primes in a row; also for  $m = n - 7$  is obtained a polynomial which generates 19 distinct primes in a row.

## VII.

The formula  $8*m^2*n^2 + 60*m*n + 29$ , where  $m$  is positive integer.

Examples:

- : for  $m = 1$  we have the polynomial  $8*n^2 + 60*n + 29$  which generates 20 distinct primes or squares of primes in a row; also for  $m = n - 17$  is obtained the polynomial  $8*m^2 - 212*m + 1321$  which generates 22 distinct primes respectively 37 primes or squares of primes in a row.

## VIII.

The formula  $11*n^2 + (2*p - 13)*n + p$ , where  $p$  is prime.

Examples:

- : for  $p = 11$  we have the polynomial  $11*n^2 + 9*n + 11$  which generates 11 distinct primes in a row; also for  $m = n - 10$  is obtained the polynomial  $11*m^2 - 211*m + 1021$  which generates 21 distinct primes in a row;
- : for  $p = 13$  we have the polynomial  $11*n^2 + 13*n + 13$  which generates 10 distinct primes in a row; also for  $m = n - 11$  is obtained the polynomial  $11*m^2 - 427*m + 4153$  which generates 21 distinct primes in a row.

## IX.

The formula  $8*n^2 - (2*p - 2)*n - p$ , where  $p$  is prime.

Examples:

- : for  $p = 13$  we have the polynomial  $8*n^2 - 24*n - 13$  which generates 10 distinct primes in a row;
- : for  $p = 37$  we have the polynomial  $8*n^2 - 72*n - 37$  which generates also many primes in a row.

## X.

The formula  $m^2*n^2 - 57*m*n + 853$ , where  $m$  is positive integer.

Examples:

- : for  $m = 1$  and  $t = n - 11$  is obtained the polynomial  $t^2 - 79t + 1601$  which generates 40 distinct primes in a row (the same primes generated by Euler's polynomial in reversed order);
- : for  $m = 2$  and  $t = n - 5$  is obtained the polynomial  $4t^2 - 154t + 1523$  which generates 40 distinct primes in a row;
- : for  $m = 3$  and  $t = n - 3$  is obtained the polynomial  $9t^2 - 225t + 1447$  which also generates many distinct primes in a row;
- : for  $m = 4$  and  $t = n - 2$  is obtained the polynomial  $16t^2 - 292t + 1373$  which generates 31 distinct primes in a row;
- : for  $m = 5$  and  $t = n - 18$  is obtained the polynomial  $25t^2 - 1185t + 14083$  which generates 32 distinct primes in a row;
- : for  $m = 9$  and  $t = n - 5$  is obtained the polynomial  $81t^2 - 1323t + 5443$  which generates 28 distinct primes in a row.

#### **XI.**

The formula  $m^2n^2 - 69mn + 1231$ , where  $m$  is positive integer.

Examples:

- : for  $m = 2$  and  $t = n - 2$  is obtained the polynomial  $4t^2 - 154t + 1523$  which generates many primes in a row;
- : for  $m = 3$  and  $t = n - 1$  is obtained the polynomial  $9t^2 - 225t + 1447$  which generates many primes in a row;
- : for  $m = 4$  and  $t = n - 12$  is obtained the polynomial  $16t^2 - 628t + 6203$  which generates 30 distinct primes in a row;
- : for  $m = 9$  and  $t = n - 15$  is obtained the polynomial  $81t^2 - 3051t + 28771$  which generates 28 distinct primes in a row.

#### **XII.**

The formula  $m^2n^2 - 149mn + 5591$ , where  $m$  is positive integer.

#### **XIII.**

The formula  $m^2n^2 - 157mn + 6203$ , where  $m$  is positive integer.

#### **XIV.**

The formula  $m^2n^2 - 77mn + 1523$ , where  $m$  is positive integer.

### **XV.**

The formula  $2*m^2*n^2 - 60*m*n + 251$ , where  $m$  is positive integer.

### **XVI.**

The formula  $2*m^2*n^2 - 140*m*n + 2251$ , where  $m$  is positive integer.

### **XVII.**

The formula  $2*(m*n + m + 1)^2 - 199$ , where  $m$  is positive integer.

Examples:

- : for  $m = 1$  is obtained the polynomial  $2*n^2 + 8*n - 191$  which generates 26 distinct primes in a row;
- : for  $m = 2$  is obtained the polynomial  $8*n^2 + 24*n - 181$  which generates 30 distinct primes in a row;
- : for  $m = 4$  and  $t = n - 6$  is obtained the polynomial  $32*n^2 - 944*n + 6763$  which generates 31 distinct primes in a row.

Note:

In this paper I considered to be primes the number 1 and the negative integers which are primes in absolute value.

This book brings together fifty-two papers regarding primes and Fermat pseudoprimes, submitted by the author to the scientific database Research Gate. Part One of this book, “Sequences of primes and conjectures on them”, contains papers on sequences of primes, squares of primes, semiprimes, pairs, triplets and quadruplets of primes and conjectures on them. This part also contains papers on possible methods to obtain large primes, some of them based on concatenation, some of them on other arithmetical operations. It is also introduced a new notion, “Smarandache-Coman sequences of primes”, defined as “all sequences of primes obtained from the Smarandache sequences using any arithmetical operation”. Part Two of this book, “Sequences of Fermat pseudoprimes and conjecture on them”, contains sequences of Poulet and Carmichael numbers. Among these papers there is a list of thirty-six polynomials and formulas that generate sequences of Fermat pseudoprimes. Part Three of this book, “Prime producing quadratic polynomials”, contains three papers which list few already known such polynomials, that generate more than 20, 30 or even 40 primes in a row, and few such polynomials discovered by the author himself (in a review of records in the field of prime generating polynomials, written by Dress and Landreau, two mathematicians well known for their contributions in this field, the author is mentioned with 18 prime producing quadratic polynomials). One of these three papers proposes 17 generic formulas that may generate prime producing quadratic polynomials.

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